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Four-point correlator constraints on electromagnetic chiral parameters and resonance effective lagrangians

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Abstract: We pursue the analysis of a set of generalized DGMLY sum rules for the electromagnetic chiral parameters at order e^2p^2 and discuss implications for effective lagrangians with resonances. We exploit a formalism in which charge spurions are introduced and treated as sources. We show that no inconsistency arises from anomalies up to quadratic order in the spurions. We focus on the sum rules associated with QCD 4-point correlators which were not analyzed in detail before. Convergence properties of the sum rules are deduced from a general analysis of the form of the counterterms in the presence of electromagnetic spurions. Following the approach in which vector and axial-vector resonances are described with antisymmetric tensor fields and have a chiral order, we show that the convergence constraints are violated at chiral order four and can be satisfied by introducing a set of terms of order six. The relevant couplings get completely and uniquely determined from a set of generalized Weinberg sum-rule relations. An update on the corrections to Dashen's low-energy theorem is given.

Keywords: Electromagnetic Processes and Properties, Sum Rules, Phenomenological Models, Chiral Lagrangians.

Contents

B. $b_1(1235)$ contributions 37

1. Motivation

Virtual photons play an important role in low energy QCD for isospin breaking phenomena and must also be taken into account in connection with precision experiments. The formalism for extending the framework of the standard ChPT [1, 2, 3] to include virtual photons at one-loop was developed, some time ago, by Urech [4]. Several extensions were later performed to accommodate leptons [5], to the weak non-leptonic sector [6], to the anomalous sector [7] as well as to the baryon sector [8]. The problem of the determination of the new set of low-energy coupling constants (LEC's) which appear in Urech's lagrangian is of obvious practical importance in relation with radiative correction calculations. Many such calculations have been performed recently: low energy $\pi\pi$ or πK scattering [9]–[15] pionic as well as pi-kaonic atoms [16, 17, 18], the set of semi-leptonic decays of the Kaon [19, 20], processes from the anomalous sector [21, 7], the problem of computing the hadronic contribution of the muon $g-2$ from τ decay data [22], the theoretical prediction of the CP-violation ratio ϵ'/ϵ for the Kaon [23, 24, 25] to mention just a few illustrative examples. Another important application of the electromagnetic low-energy couplings (LEC) is to the question of the chiral corrections to Dashen's low-energy theorem [26] and the determination of the light quark masses (e.g. [27]). Interesting questions were raised recently concerning the proper definition of the quark masses in the presence of QED [28].

The problem of the determination of the EM LEC's was addressed in several papers some time ago [29, 30, 31] but the results remain incomplete and sometimes contradictory. In this paper, we continue the discussion started in ref. [31] (hereafter referred to as (I)). In (I) it was shown that the electromagnetic LEC's obey integral sum rule representations which generalize the classic DGMLY sum rule [32] for the $\pi^+ - \pi^0$ mass difference. The integral representations have the form of convolutions involving a pure QCD n-point Green's function with $n = 2, 3$ and 4 together with the free photon propagator. These representations serve a number of purposes. For instance, they can be used to recover the chiral scale dependence of the LEC's, to determine how these depend on the ξ gauge parameter and, also, by studying the convergence properties, to determine which LEC's are affected by short distance ambiguities. Finally, these representations can be used as a method to provide approximate determinations of these couplings.

This was exploited in (I) to discuss the set of couplings which are associated with 2- and 3-point Green's functions. The method consists in constructing rational approximants to the Green's functions, which is in the spirit of the large N_c expansion [33]. The parameters of the rational functions are constrained by the chiral Ward identities, by physical input on resonance masses and couplings and by short-distance matching conditions. In the present paper, we address the more complicated case of the 4-point functions. The way of insuring that the Ward identities are satisfied is to start from a chiral lagrangian including resonances. Such a lagrangian was constructed in ref. [34] which uses the antisymmetric tensor formalism [2] to describe the vector and axial-vector resonances. This formalism is particularly convenient for discussing the ordinary LEC's because one can assign a chiral order to each resonance field such that all the terms which are of order four, which include one multiplet of vector, axial-vector, scalar and pseudo-scalar resonances were listed in ref. [34]. It was shown in a subsequent paper [35] that if one imposes a number of short distance matching conditions, the determination of the $O(p^4)$ LEC's becomes independent of the specific representation of the resonances in the chiral lagrangian.

The lagrangian of ref. [34] was employed by Baur and Urech [29] (BU) to discuss the electromagnetic LEC's. For this application, however, one realizes by using the integral representation that the Green's functions are needed under kinematical conditions which are different, in general, from those considered in [35] such that the lagrangian of ref. [34] is no longer sufficient to ensure the proper short distance constraints. This deficiency manifests itself in the fact that the results of BU for the electromagnetic LEC's fail to satisfy the correct scale dependence, which is an effect of leading order in N_c. In this paper, we will show that a simple extension of the lagrangian [34] to include a set of terms which are of chiral order six allows one to satisfy the short distance constraints in the chiral limit which are necessary for a consistent determination of the electromagnetic LEC's and which are associated with the 2- and 3-point functions considered in [31] as well as the 4-point functions introduced in this paper. Finally, we will present estimates for the six LEC's which are associated with these 4-point functions.

2. Chiral calculation of the basic correlators

2.1 Definition of the basic correlation functions

In the presence of a dynamical photon field F_μ it is convenient to extend the source part of the QCD lagrangian to include two spurion fields q_V, q_A [4]

$$\mathcal{L}_{\rm QCD}^{\rm sources} = \bar\psi_L \gamma^\mu [l_\mu + q_L\, F_\mu]\psi_L + \bar\psi_R \gamma^\mu [r_\mu + q_R\, F_\mu]\psi_R - \bar\psi(s - i\gamma^5 p)\psi\,, \qquad (2.1)$$

with

$$l_\mu = v_\mu - a_\mu\,, \qquad q_L = q_V - q_A\,, \qquad r_\mu = v_\mu + a_\mu\,, \qquad q_R = q_V + q_A\,. \qquad (2.2)$$

The number of light flavours is assumed to be $N_F^0 = 3$, and the sources in eq. (2.2) are traceless 3×3 matrices. Originally [4], the spurions q_V, q_A were introduced only for the purpose of classifying the independent terms in the chiral lagrangian and afterwards one would set $q_A = 0$, $q_V = Q$. We will exploit here more general applications by considering the spurions on the same footing as the ordinary external sources. We note that renormalization will generate terms which are non-linear in the sources. Those which are quadratic in the spurion sources are of particular interest for our purposes and will be discussed in detail in section 4. It will also be argued that no difficulty arises in the definition of the generating functional in the presence of the q_V, q_A spurions up to quadratic order in the spurions. We now introduce a set of five correlation functions defined by taking two functional derivatives

JHEP06(2004)047

with respect to v_μ or a_μ and two derivatives with respect to the charge spurions q_V or q_A. The first one is defined as follows

$$1. \quad \langle A^a_\alpha A^b_\beta Q^c_A Q^d_A \rangle = \int d^4x \, d^4y \, d^4z \exp(iky) \frac{\delta^4 \, W(v_\mu, a_\mu, q_V, q_A)}{\delta a^a_\alpha(x) \delta a^b_\beta(y) \delta q^c_A(z) \delta q^d_A(0)} \tag{2.3}$$

where W is the generating functional of connected Green's functions. The remaining four

$$\begin{aligned} &2. \ \langle A^a_\alpha A^b_\beta Q^c_V Q^d_V \rangle \\ &3. \ \langle V^a_\alpha V^b_\beta Q^c_A Q^d_A \rangle \\ &4. \ \langle V^a_\alpha V^b_\beta Q^c_V Q^d_V \rangle \\ &5. \ \langle A^a_\alpha V^b_\beta Q^c_A Q^d_V \rangle \end{aligned} \tag{2.4}$$

are defined in an exactly analogous way. These correlators involve two Lorentz structures $g_{\alpha\beta}$ and $k_\alpha k_\beta$. For our purposes, we can restrict ourselves to the Lorentz structure proportional to $g_{\alpha\beta}$ and we will consider the limit $k \to 0$. Before taking this limit, however, we must consider two possible kinds of singularities: a) $1/k^2$ pion poles and b) $\log(k^2)$ cuts due to pion loops. The latter singularities drop out in the leading large N_c approximation, which we will frequently invoke. A pion pole can appear only in one flavour structure of the fifth correlator $\langle A^a_\alpha V^b_\beta Q^c_A Q^d_V \rangle$. Apart from this particular case, we can simply set $k = 0$, meaning that we restrict ourselves to constant vector and axial-vector sources. This leads to great simplification since all terms like $\partial_\mu v_\nu$, $\partial_\mu a_\nu$ etc... can be dropped in the chiral resonance lagrangian. Similarly, the q_V and q_A sources can be taken as constant for the present applications.

From the definition, eq. (2.3) and the explicit expression of the terms linear in the sources eq. (2.1) we can easily see that the correlators introduced above can be represented as a convolution of an ordinary connected QCD 4-point Green's function and the photon propagator. For example, one finds

$$\langle A^a_\alpha A^b_\beta Q^c_A Q^d_A \rangle = \int d^4x d^4y d^4z \exp(iky) \langle 0 | T(A^a_\alpha(x) A^b_\beta(y) A^c_\rho(z) A^d_\sigma(0)) | 0 \rangle D^{\rho\sigma}_F(z) \,. \tag{2.5}$$

Because of the short distance singularities, the integral over d^4z is expected to be divergent. The divergence is removed upon including the additional contributions stemming from the counterterms and one may use d-dimensional integration as a regularization method.

Let us now examine the flavour structure. The correlators depend on four flavour indices. At order e^2p^2 one can show by using the chiral lagrangian that there are only four independent tensor structures. It will prove convenient to use the following basis

$$\begin{aligned} \mathbf{e_1} &= f^{tac} f^{tbd} + f^{tad} f^{tbc} \\ \mathbf{e_2} &= \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc} \\ \mathbf{e_3} &= d^{tac} d^{tbd} + d^{tad} d^{tbc} \\ \mathbf{e_4} &= f^{tab} f^{tcd} \,. \end{aligned} \tag{2.6}$$

Other tensor structures that arise in practical calculations can be expressed in terms of this basis by utilizing SU(3) trace identities. The following relations are useful

$$\begin{aligned} d^{tab}d^{tcd} &= \frac{1}{6}\mathbf{e}_1 + \frac{1}{3}\mathbf{e}_2 - \frac{1}{2}\mathbf{e}_3 \\ \delta^{ab}\delta^{cd} &= \frac{1}{2}\mathbf{e}_1 + \frac{3}{2}\mathbf{e}_3 \,. \end{aligned} \tag{2.7}$$

We can now expand the correlators over the flavour basis. The first four correlators are expanded over $\mathbf{e_1}$, $\mathbf{e_2}$, $\mathbf{e_3}$ because of the permutation symmetry $a \leftrightarrow b$, $c \leftrightarrow d$. For instance, one has

$$\langle A^a_\alpha A^b_\beta Q^c_A Q^d_A \rangle = i g_{\alpha\beta} \sum_{j=1}^{3} \langle AAQ_AQ_A\rangle_j \mathbf{e_j} \,. \tag{2.8}$$

The expansion of the last correlator involves $\mathbf{e_4}$ also,

$$\langle A^a_\alpha V^b_\beta Q^c_A Q^d_V \rangle = i g_{\alpha\beta} \sum_{j=1}^{4} \langle AVQ_AQ_V\rangle_j \mathbf{e_j} \,. \tag{2.9}$$

This expansion apparently generates sixteen independent flavour components. As we will see below there are only ten components which appear at the chiral order e^2p^2 considered here.

2.2 Correlators at order e^2p^2

We start by computing the correlators defined above (2.3) (2.4) in the chiral expansion up to the order e^2p^2. Setting the sources s and p equal to zero, the leading order lagrangian collects the terms of order e^2 and p^2

$$\begin{aligned} \mathcal{L}^{(2)}_{\text{chir}} &= \mathcal{L}^{p^2}_{\text{chir}} + \mathcal{L}^{e^2}_{\text{chir}} \\ \mathcal{L}^{p^2}_{\text{chir}} &= \frac{F_0^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu F^\mu)^2 + \frac{1}{2}M_\gamma^2 F_\mu F^\mu \\ \mathcal{L}^{e^2}_{\text{chir}} &= C\langle q_L U^\dagger q_R U\rangle \,, \end{aligned} \tag{2.10}$$

with

$$D_\mu U = \partial_\mu U - i\left(r_\mu + q_R F_\mu\right) U + iU\left(l_\mu + q_L F_\mu\right). \tag{2.11}$$

The photon is endowed with a small mass (which may be counted as $O(p)$) in order to regulate infrared divergences which can appear in the chiral limit. We will also need the following part of the chiral lagrangian of order e^2p^2 [4]

$$\begin{aligned} \mathcal{L}^{e^2p^2}_{\text{chir}} = &\frac{1}{2}K_1 F_0^2 \langle D_\mu U D^\mu U^\dagger\rangle\langle q_L q_L + q_R q_R\rangle + \\ &+K_2 F_0^2\langle D_\mu U D^\mu U^\dagger\rangle\langle q_L U^\dagger q_R U\rangle + \\ &+K_3 F_0^2(\langle D_\mu U q_L U^\dagger\rangle^2 + \langle D_\mu U^\dagger q_R U\rangle^2) + \\ &+K_4 F_0^2\langle D_\mu U q_L U^\dagger\rangle\langle D_\mu U^\dagger q_R U\rangle + \\ &+K_5 F_0^2\langle D_\mu U^\dagger D^\mu U q_L q_L + D_\mu U D^\mu U^\dagger q_R q_R\rangle + \end{aligned}$$

$$
\begin{aligned}
&+K_6F_0^2\langle D_\mu U^\dagger D^\mu U q_L U^\dagger q_R U + D_\mu U D^\mu U^\dagger q_R U q_L U^\dagger\rangle + \\
&+K_{12}F_0^2\langle U D^\mu U^\dagger[\nabla_\mu q_R, q_R] + U^\dagger D^\mu U[\nabla_\mu q_L, q_L]\rangle + \\
&+K_{13}F_0^2\langle \nabla_\mu q_R\, U \nabla^\mu q_L\, U^\dagger\rangle + \\
&+K_{14}F_0^2\langle \nabla_\mu q_L \nabla^\mu q_L + \nabla_\mu q_R \nabla^\mu q_R\rangle
\end{aligned} \tag{2.12}
$$

where

$$
\nabla_\mu q_L = \partial_\mu q_L - i[l_\mu, q_L]\,, \qquad \nabla_\mu q_R = \partial_\mu q_R - i[l_\mu, q_R]\,. \tag{2.13}
$$

As usual, the correlators collect the contributions at one-loop generated from $\mathcal{L}^{(2)}_{\text{chir}}$ and the tree contributions from $\mathcal{L}^{e^2p^2}_{\text{chir}}$. Let us first quote the results, some details about how the calculation proceeds will be given later. For the components of $\langle AAQ_AQ_A\rangle$ one obtains

$$
\begin{aligned}
\langle AAQ_AQ_A\rangle_1 &= F_0^2\Big[2(K_1^r-K_2^r)+2(K_5^r-K_6^r)+4K_{12}^r+K_{13}^r+2K_{14}^r+ \\
&\qquad + \chi_1(M_\gamma,\mu)+\frac{5}{4}Z(k,\mu)\Big] \\
\langle AAQ_AQ_A\rangle_2 &= F_0^2\Big[-2(2K_3^r+K_4^r)+\frac{4}{3}(K_5^r-K_6^r)+\frac{3}{2}Z(k,\mu)\Big] \\
\langle AAQ_AQ_A\rangle_3 &= F_0^2\Big[6(K_1^r-K_2^r)+2(K_5^r-K_6^r)+\frac{9}{4}Z(k,\mu)\Big]
\end{aligned} \tag{2.14}
$$

where K_i^r stand for the renormalized scale dependent parameters. The function $\chi_1(M_\gamma,\mu)$ is one of χ_i, $i=1,2,3,4$ which are the one-loop chiral contributions involving a virtual photon. We will describe the computation in detail in the next section. The contributions proportional to the function $Z(k,\mu)$ arise from purely pionic one-loop diagrams having one vertex from the term proportional to C in eq. (2.10). This function reads

$$
Z(k,\mu) = \frac{2C}{16\pi^2F_0^4}\left(-\log\frac{-k^2}{\mu^2}+1\right)\,. \tag{2.15}
$$

While we include this contribution here for completeness we note that it is of subleading order in the large N_c expansion. In the following, we will drop such contributions for consistency with the leading large N_c approximation which we will make in the treatment of the resonances. We also note that the correlators must be independent of the chiral renormalization scale μ. This constraint allows one to recover the scale dependence of $K_i^r(\mu)$ computed in refs. [4, 37], generalized to an arbitrary ξ gauge. The components of the second correlator has the following expression

$$
\begin{aligned}
\langle AAQ_VQ_V\rangle_1 &= F_0^2\Big[2(K_1^r+K_2^r)+2(K_5^r+K_6^r)+4K_{12}^r-K_{13}^r+2K_{14}^r+ \\
&\qquad + \chi_2(M_\gamma,\mu)-\frac{5}{4}Z(k,\mu)\Big] \\
\langle AAQ_VQ_V\rangle_2 &= F_0^2\Big[-2(2K_3^r-K_4^r)+\frac{4}{3}(K_5^r+K_6^r)-\frac{3}{2}Z(k,\mu)\Big] \\
\langle AAQ_VQ_V\rangle_3 &= F_0^2\Big[6(K_1^r+K_2^r)+2(K_5^r+K_6^r)-\frac{9}{4}Z(k,\mu)\Big]\,.
\end{aligned} \tag{2.16}
$$

For the next two correlators, only the first flavour structure appears at this chiral order

$$\begin{aligned}\langle VVQ_AQ_A\rangle_1 &= F_0^2[-K_{13}^r + 2K_{14}^r + \chi_3(M_\gamma,\mu)]\\ \langle VVQ_VQ_V\rangle_1 &= F_0^2[K_{13}^r + 2K_{14}^r]\,.\end{aligned} \tag{2.17}$$

The last correlator has two non-vanishing flavour structures. The structure proportional to $\mathbf{e_4}$ is the only one to receive a tree-level contribution from $\mathcal{L}^{(2)}_{\text{chir}}$,

$$\begin{aligned}\langle AVQ_AQ_V\rangle_1 &= F_0^2\,[2K_{12}^r + 2K_{14}^r + \chi_4(M_\gamma,\mu)]\\ \langle AVQ_AQ_V\rangle_4 &= \frac{2C}{k^2} + F_0^2\,[-K_{13}^r + 2K_{12}^r + \chi_4(M_\gamma,\mu)]\,.\end{aligned} \tag{2.18}$$

We now introduce a set of combinations of these correlator components which enjoy useful properties

$$\begin{aligned}\Pi_1 &= \langle AAQ_AQ_A\rangle_1 - 2\langle AVQ_AQ_V\rangle_1 + \langle VVQ_AQ_A\rangle_1\\ \Pi_2 &= \langle AAQ_VQ_V\rangle_1 - 2\langle AVQ_AQ_V\rangle_1 + \langle VVQ_VQ_V\rangle_1\\ \Pi_3 &= \langle AAQ_AQ_A\rangle_3\\ \Pi_4 &= \langle AAQ_VQ_V\rangle_3\\ \Pi_5 &= \langle AAQ_AQ_A\rangle_2 - \frac{2}{3}\langle AAQ_AQ_A\rangle_3\\ \Pi_6 &= \langle AAQ_VQ_V\rangle_2 - \frac{2}{3}\langle AAQ_VQ_V\rangle_3\,.\end{aligned} \tag{2.19}$$

These six combinations are in one to one correspondence with the parameters $K_1^r,\ldots,K_6^r$ which we intend to determine. One can show (see [31]) that they are independent of the gauge parameter ξ, and we will show below that they get no contributions from the QCD counterterms. This last fact implies that the photon loop integral must be converging. Convergence must be understood in the sense that integration is to be performed in d dimensions and the limit $d \to 4$ is well defined.

In the leading large N_c approximation one has

$$\Pi_5 = \Pi_6 = 0 \qquad (\text{large } N_c)\,. \tag{2.20}$$

This is not difficult to show. The graphs which are of leading order in N_c are the planar ones which involve a single fermion loop [33]. This implies that they involve a single trace over the flavour λ matrices. Furthermore, we are interested in four-point QCD correlators which are symmetric under permutation of the Lorentz indices α, β as well as in the other two indices $\rho\sigma$ (see eq. (2.5)). They must then have the following structure

$$\begin{aligned}\langle A^a_\alpha A^b_\beta Q^c_A Q^d_A\rangle &= ig_{\alpha\beta}\left(E\langle\{\lambda^a,\lambda^b\}\{\lambda^c,\lambda^d\}\rangle + F\langle\lambda^a\lambda^c\lambda^b\lambda^d + \lambda^a\lambda^d\lambda^b\lambda^c\rangle\right)\\ &= ig_{\alpha\beta}\left(2(2E-F)\,\mathbf{e_1} + \frac{4}{3}(2E+F)\left(\mathbf{e_2} + \frac{3}{2}\mathbf{e_3}\right)\right)\end{aligned} \tag{2.21}$$

making use of relations (2.7), where E and F are arbitrary, and the same holds for the propagator with Q_AQ_A replaced by Q_VQ_V. This proves relations (2.20). From these, one can recover the large N_c relations among the corresponding K_i^r parameters

$$K_3^r = -K_1^r \quad K_4^r = 2K_2^r \qquad (\text{large } N_c) \tag{2.22}$$

noted in refs. [4] and [30]. We will further consider the following three combinations

$$\begin{aligned}\Pi_7 &= \langle VVQ_VQ_V\rangle_1 - \langle VVQ_AQ_A\rangle_1\\ \Pi_8 &= \langle AVQ_AQ_V\rangle_1 - \langle VVQ_VQ_V\rangle_1\\ \Pi_9 &= \lim_{k^2\to 0} k^2\langle AVQ_AQ_V\rangle_4\,.\end{aligned} \tag{2.23}$$

From these combinations we will recover established results concerning the couplings C, K_{12}^r, K_{13}^r. Π_7 and Π_8 depend on the gauge. Convergence of the photon loop integral can be established for Π_7 and Π_9 while for Π_8 it holds in the particular gauge $\xi = 0$ and in a limited sense, as we will discuss in more detail. This limitation affects the determination of K_{12}^r. We will not attempt to estimate the coupling K_{14}^r which is a pure source term.

One further remark is in order. One usually expects the coupling constants in ChPT to be associated with QCD correlators which are order parameters, except for the pure source terms. Looking at eqs. (2.19) this, at first sight, seems sometimes not to be the case here. For instance, Π_3 is associated with the correlator $\langle A_\alpha^a A_\beta^b A_\mu^c A_\nu^d\rangle$ which does not vanish in perturbation theory. However, this is simply due to the fact that the components $\langle AVQ_AQ_V\rangle_3$ and $\langle VVQ_VQ_V\rangle_3$ vanish at chiral order e^2p^2, so one could equally well express Π_3 in terms of a correlator which does behave as an order parameter.

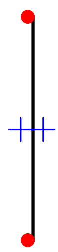
2.3 Calculation of the one-loop photon contributions

In this section, we will compute contributions to the generating functional $W(v_\mu, a_\mu, q_V, q_A)$ arising from the chiral lagrangian $\mathcal{L}_{\rm chir}^{(2)}$. The use of the spurions q_V, q_A as sources gives rise to a simple diagrammatic method which we will in this section and the following ones. As there were some errors reported in related calculations in the past (see e.g. [36]) and we will, in fact, find some discrepancies with BU, we will present our calculations in some detail. Here, we will compute the functions $\chi_i(M_\gamma,\mu)$ which appear in eqs. (2.16). They arise from sets of one-loop diagrams containing one photon line and one pion line, the vertices being obtained from the leading order lagrangian (2.10). The corresponding standard ξ-gauge massive photon propagator reads

$$D_F^{\mu\nu}(p) = \frac{-i}{(p^2-M_\gamma^2)}\left\{g^{\mu\nu} + (\xi-1)\frac{p^\mu p^\nu}{(p^2-\xi M_\gamma^2)}\right\}. \tag{2.24}$$

The list of the vertices which are effectively needed is displayed in the appendix. We remark here that these contributions are to be included both in the chiral expansion of the correlator as well as in the expression from the resonance lagrangian since the latter includes the piece from eq. (2.10).

Let us now compute the contribution to $\langle AAQ_AQ_A\rangle_i$. A single diagram contributes, shown below



The following conventions are used to represent the sources

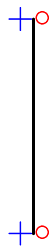
● = q_A source   + = a_μ source

○ = q_V source   x = v_μ source

The relevant vertices are listed in the appendix. One must keep in mind, for instance, that the coupling of one axial source to the pion does not contribute to our correlators. The photon line which joins the two q-sources is not explicitly drawn. Each diagram is easy to interpret in terms of the integral representation eq. (2.5). We will first give the results in integral form, in order to display the integrands, in which we perform the replacement of $p_\alpha p_\beta$ by $p^2 g_{\alpha\beta}/d$. In integral form, this diagrams gives

$$\langle AAQ_AQ_A\rangle_1 = -F_0^2 \int \frac{-i\, d^dp}{(2\pi)^d} \frac{\xi}{p^2(p^2-\xi M_\gamma^2)} \tag{2.25}$$

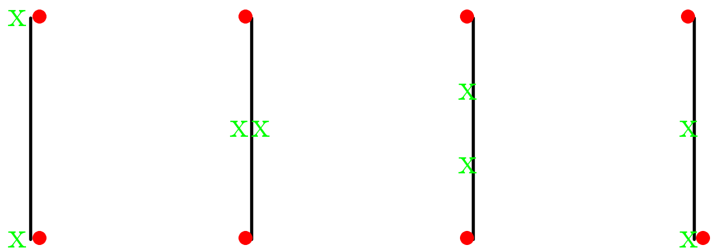
and the other components $\langle AAQ_AQ_A\rangle_i$ get no contributions. Next, we consider $\langle VVQ_VQ_V\rangle_1$ and, again, there is a single diagram to compute shown below



which gives

$$\langle AAQ_VQ_V\rangle_1 = -F_0^2 \int \frac{-i\, d^dp}{(2\pi)^d} \left\{ \frac{1}{p^2(p^2-M_\gamma^2)} + \frac{1}{d}\frac{\xi-1}{(p^2-M_\gamma^2)(p^2-\xi M_\gamma^2)} \right\} \tag{2.26}$$

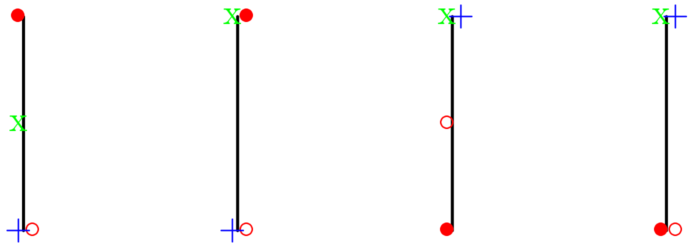
and the other components vanish. One also finds that $\langle VVQ_VQ_V\rangle_i$ receive no contribution from pions. Next we consider $\langle VVQ_AQ_A\rangle_i$, in this case there are four diagrams



Calculating these diagrams, one finds that the last two cancel each other and the first two give the following integral

$$\langle VVQ_AQ_A\rangle_1 = F_0^2 \left(1-\frac{1}{d}\right) \int \frac{-i\, d^dp}{(2\pi)^d} \frac{\xi-1}{(p^2-M_\gamma^2)(p^2-\xi M_\gamma^2)}\,. \tag{2.27}$$

Finally, we have to compute $\langle AVQ_AQ_V\rangle_i$. There are four diagrams in this case



Computing these diagrams gives

$$
\begin{aligned}
\langle AVQ_AQ_V\rangle_1 &= -F_0^2\int\frac{-i\,d^dp}{(2\pi)^d}\left\{\frac{1}{d}\frac{\xi}{p^2(p^2-\xi M_\gamma^2)}-\frac{1}{2}\frac{1}{p^2(p^2-M_\gamma^2)}-\right.\\
&\qquad\left.-\frac{1}{2d}\frac{\xi-1}{(p^2-M_\gamma^2)(p^2-\xi M_\gamma^2)}\right\}\\
\langle AVQ_AQ_V\rangle_4 &= \frac{F_0^2(1-d)}{k^2}\int\frac{-i\,d^dp}{(2\pi)^d}\frac{1}{p^2}+\langle AVQ_AQ_V\rangle_1\,. \qquad (2.28)
\end{aligned}
$$

The $1/k^2$ term is generated by the third and fourth diagrams and the integral vanishes in dimensional regularization. We can now compute the d-dimensional integrals and the divergences are removed using the chiral $\overline{MS}$ prescription [3]. The results for the loop functions $\chi_i(M_\gamma,\mu)$ are collected below

$$
\begin{aligned}
\chi_1(M_\gamma,\mu) &= \frac{1}{16\pi^2}\left[\xi\log\frac{M_\gamma^2}{\mu^2}+\xi\log\xi\right]\\
\chi_2(M_\gamma,\mu) &= \frac{1}{16\pi^2}\left(\frac{1}{8}\right)\left[2(\xi+3)\log\frac{M_\gamma^2}{\mu^2}+2\xi\log\xi-\xi+1\right]\\
\chi_3(M_\gamma,\mu) &= \frac{1}{16\pi^2}\left(-\frac{1}{8}\right)\left[6(\xi-1)\log\frac{M_\gamma^2}{\mu^2}+6\xi\log\xi+\xi-1\right]\\
\chi_4(M_\gamma,\mu) &= \frac{1}{16\pi^2}\left(\frac{1}{16}\right)\left[2(\xi-3)\log\frac{M_\gamma^2}{\mu^2}+2\xi\log\xi-\xi-1\right]. \qquad (2.29)
\end{aligned}
$$

Using these results and those from the preceding section which display the contributions involving the chiral couplings K_i^r we can deduce the scale and gauge dependence of the latter. One finds that $K_1^r,\ldots,K_6^r$ are independent of the gauge parameter ξ and the scale dependence is in agreement with the results of [4, 37]. The couplings K_{12}^r, K_{13}^r, K_{14}^r do depend on ξ. Defining the beta functions Σ_i in the conventional manner

$$
\frac{dK_i^r(\mu)}{d\mu}\equiv-\frac{1}{16\pi^2}\,\Sigma_i \qquad (2.30)
$$

one finds from eqs. (2.29)

$$
\Sigma_{12}=\frac{1}{4}(3-2\xi),\qquad \Sigma_{13}=\frac{3}{4}(1-\xi),\qquad \Sigma_{14}=-\frac{3}{8}(1-\xi)\,. \qquad (2.31)
$$

These results agree with those of Urech in the special case $\xi=1$, and they reproduce the results presented in ref. [31] where a different set of correlation functions were employed.

3. Correlators from the minimal resonance lagrangian

3.1 $O(p^4)$ resonance lagrangian

Our starting point for determining the K_i^r parameters is the integral representation eq. (2.5) in terms of 4-point correlation functions. The first approximation that we make is to consider the leading large N_c limit. In this limit, it is an exact statement that QCD correlators can be expressed as a sum over tree graphs involving resonance propagators [33] (see e.g. [38] for a review of recent applications). The second approximation that we make is to restrict ourselves to contributions from a finite number of resonances. This is acceptable in our case because we consider combinations of correlators which fall off as powers for large momenta.

In order to guarantee the validity of chiral Ward identities one may compute the correlators starting from a chiral lagrangian with resonances. In cases where the integrals can be shown to converge, imposing the convergence property generates a set of constraints for the parameters of the resonance lagrangian which we will call consistency conditions. These conditions will be discussed in detail in the following. We begin with the resonance lagrangian which was used by Baur and Urech [29]

$$\mathcal{L}^{(4)}_{\rm res} = \mathcal{L}^{p^2}_{\rm chir} + \mathcal{L}^{(4)}_{S,S_0} + \mathcal{L}^{(4)}_{V,A}\,. \tag{3.1}$$

BU have used exactly the lagrangian proposed by Ecker et al. [34] (except for leaving out the π' resonance multiplet) which we will call the minimal resonance lagrangian. Let us recall the notation and formalism used in ref. [34]. The formalism that is used is one in which all resonances transform homogeneously under the non linear representation of the chiral group including the vector and the axial-vector resonances which are described as antisymmetric tensor fields. In this approach, all the resonance fields can be assigned a chiral order equal to two, such that the resonance lagrangian of ref. [34], contains all possible terms, for each resonance field, which are of chiral order four. The only terms of chiral order six which are present are the kinetic energy terms.

We recall here the details: in the scalar sector an octet S and a singlet S_0 are considered and the lagrangian involves two coupling-constants, c_d and $\tilde{c}_d$,

$$\begin{aligned}\mathcal{L}_{S,S_0} =\; & \frac{1}{2}\langle \nabla^\lambda S \nabla_\lambda S - M_S^2 S^2\rangle + \frac{1}{2}[\partial^\lambda S_0 \partial_\lambda S_0 - M_{S_0}^2 S_0^2] + \\ & + c_d \langle S u_\mu u^\mu \rangle + \tilde{c}_d S_0 \langle u_\mu u^\mu\rangle\,.\end{aligned} \tag{3.2}$$

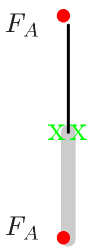
Next, a nonet of vector and axial-vector resonances are considered and the minimal lagrangian involves three coupling constants F_A, F_V and G_V,

$$\begin{aligned}\mathcal{L}_{V,A} =\; & -\frac{1}{2}\sum_{R=V,A} \langle \nabla^\lambda R_{\lambda\mu}\nabla_\nu R^{\nu\mu} - \frac{1}{2} M_R^2 R_{\mu\nu}R^{\mu\nu}\rangle + \\ & + \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle + \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle\,.\end{aligned} \tag{3.3}$$

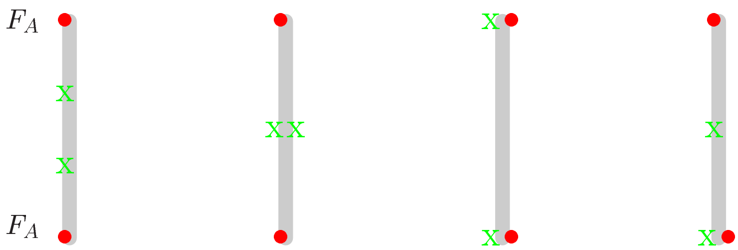
We will compute the basic correlators introduced in section1 first from this minimal resonance lagrangian. It is convenient to keep using the q_V, q_A source spurions at this level, introduced in association with the v_μ and a_μ sources, with the replacements $l_\mu \to l_\mu + q_L F_\mu$, $r_\mu \to r_\mu + q_R F_\mu$. The diagrammatic calculation is exactly equivalent to using the convolution representation eq. (2.5) in which the QCD correlator is computed from the resonance lagrangian. In principle, we should be able to reproduce the results of BU (who used a completely different approach to determine the K_i^r's). We follow their convention to regularize the UV divergences in the chiral $\overline{MS}$ prescription and we introduce a regularization scale Λ. The various vertices needed in the calculation are collected in the appendix. We will use the photon propagator in a general ξ gauge as in eq. (2.24). Since the resonances are massive one can set the photon mass $M_\gamma = 0$ in the following. Since the $O(p^2)$ chiral lagrangian is also part of the resonance lagrangian, the pion contributions $\chi_i(M_\gamma, \Lambda)$, $i = 1, 2, 3, 4$ which we already computed (eqs. (2.29)) must be added to the resonance contributions.

3.2 Contributions proportional to F_A^2 and F_V^2

We will now consider successively the contributions proportional to F_A^2, F_V^2, $G_V F_V$, G_V^2, $G_V^2 F_V^2$ which are the only ones generated from the vector and axial-vector part of the resonance lagrangian eq. (3.1). One easily sees, at first, that the following correlator components $\langle AAQ_AQ_A\rangle_i$ and $\langle VVQ_VQ_V\rangle_1$ get no contributions proportional to F_A^2. Next, one finds that there are five diagrams which contribute to $\langle VVQ_AQ_A\rangle_i$, the relevant vertices can be found in appendix A. The diagram shown below is found to make a vanishing contribution



as it is antisymmetric in the α β indices. The remaining four diagrams are



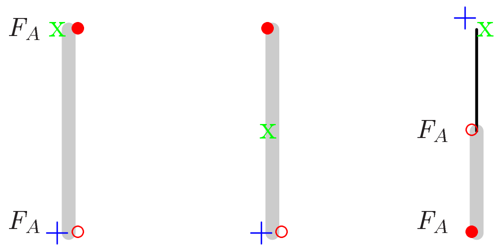
The result is of the following form

$$\langle VVQ_AQ_A\rangle_1 = F_A^2 \int \frac{-i\, d^dp}{(2\pi)^d} \sum_1^4 D_i(p)\,, \tag{3.4}$$

with the individual diagrams contributions to the integrand reading

$$
\begin{aligned}
D_1 &= \left(1-\frac{1}{d}\right)\left(\frac{2-d}{M_A^2(p^2-M_A^2)}+\frac{6-d}{(p^2-M_A^2)^2}+\frac{4M_A^2}{(p^2-M_A^2)^3}\right)\\
D_2 &= \left(1-\frac{1}{d}\right)\frac{-2}{(p^2-M_A^2)^2}\\
D_3 &= \left(1-\frac{1}{d}\right)\left(\frac{2-d}{M_A^2(p^2-M_A^2)}+\frac{d-1+\xi}{p^2(p^2-M_A^2)}\right)\\
D_4 &= \left(1-\frac{1}{d}\right)\left(\frac{2(d-2)}{M_A^2(p^2-M_A^2)}-\frac{4}{(p^2-M_A^2)^2}\right),
\end{aligned}
\tag{3.5}
$$

respectively. Next, we consider $\langle AAQ_VQ_V\rangle$. A single diagram contributes (with a crossed one) which gives the same result as D_3 above. Finally, we turn to $\langle AVQ_AQ_V\rangle$. There are three diagrams which contribute (and no crossed diagrams in this case), which are shown below.



These diagrams give

$$
\begin{aligned}
\langle AVQ_AQ_V\rangle_1 &= F_A^2\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\left[\frac{d+\xi-1}{2p^2(p^2-M_A^2)}-\frac{1}{(p^2-M_A^2)^2}\right]\\
k^2\langle AVQ_AQ_V\rangle_4 &= F_A^2\int\frac{-i\,d^dp}{(2\pi)^d}\,\frac{1-d}{p^2-M_A^2}\,.
\end{aligned}
\tag{3.6}
$$

Computing all the integrals, in summary, the following results are obtained

$$
\begin{aligned}
\langle AAQ_AQ_A\rangle_i^{F_A^2} &= 0\\
\langle AAQ_VQ_V\rangle_1^{F_A^2} &= -\frac{3}{4}\frac{F_A^2}{16\pi^2}(1+\xi)\left(\frac{1}{6}+\log\frac{M_A^2}{\Lambda^2}\right)\\
\langle VVQ_AQ_A\rangle_1^{F_A^2} &= \frac{3}{2}\frac{F_A^2}{16\pi^2}\left\{1-\frac{1}{2}(\xi-1)\left(\frac{1}{6}+\log\frac{M_A^2}{\Lambda^2}\right)\right\}\\
\langle VVQ_VQ_V\rangle_1^{F_A^2} &= 0\\
\langle AVQ_AQ_V\rangle_1^{F_A^2} &= -\frac{3}{8}\frac{F_A^2}{16\pi^2}(1+\xi)\left(\frac{1}{6}+\log\frac{M_A^2}{\Lambda^2}\right)\\
k^2\langle AVQ_AQ_V\rangle_4^{F_A^2} &= 3\frac{M_A^2F_A^2}{16\pi^2}\left(\frac{2}{3}+\log\frac{M_A^2}{\Lambda^2}\right)+O(k^2)\,.
\end{aligned}
\tag{3.7}
$$

There is a simple correspondence between the diagrams proportional to F_V^2 and those proportional to F_A^2 considered above so we give directly the results for the various components

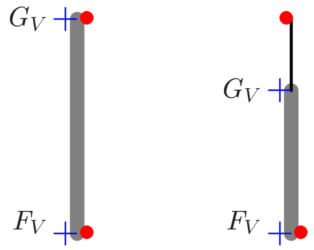
JHEP06(2004)047

of the correlators

$$
\begin{aligned}
\langle AAQ_AQ_A\rangle_1^{F_V^2} &= -\frac{3}{4}\frac{F_V^2}{16\pi^2}(1+\xi)\left(\frac{1}{6}+\log\frac{M_V^2}{\Lambda^2}\right)\\
\langle AAQ_VQ_V\rangle_i^{F_V^2} &= 0\\
\langle VVQ_AQ_A\rangle_1^{F_V^2} &= 0\\
\langle VVQ_VQ_V\rangle_1^{F_V^2} &= \frac{3}{2}\frac{F_V^2}{16\pi^2}\left\{1-\frac{1}{2}(\xi-1)\left(\frac{1}{6}+\log\frac{M_V^2}{\Lambda^2}\right)\right\}\\
\langle AVQ_AQ_V\rangle_1^{F_V^2} &= -\frac{3}{8}\frac{F_V^2}{16\pi^2}(1+\xi)\left(\frac{1}{6}+\log\frac{M_V^2}{\Lambda^2}\right)\\
k^2\langle AVQ_AQ_V\rangle_4^{F_V^2} &= -3\frac{M_V^2F_V^2}{16\pi^2}\left(\frac{2}{3}+\log\frac{M_V^2}{\Lambda^2}\right).
\end{aligned}
\tag{3.8}
$$

3.3 G_VF_V contributions

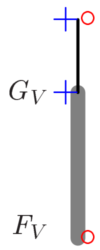
We have two diagrams which contribute to $\langle AAQ_AQ_A\rangle_1$ shown below.



This results in the following integral for the correlator

$$
\langle AAQ_AQ_A\rangle_1 = 4\,G_VF_V\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\left(\frac{d-2}{M_V^2(p^2-M_V^2)}+\frac{1-d}{p^2(p^2-M_V^2)}\right). \tag{3.9}
$$

Next we have one diagram contributing to $\langle AAQ_VQ_V\rangle_1$ which is non vanishing

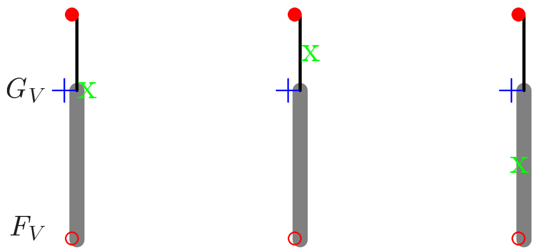


and which gives

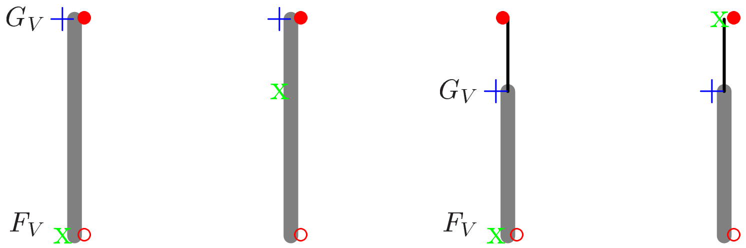
$$
\langle AAQ_VQ_V\rangle_1 = 4G_VF_V\int\frac{-i\,d^dp}{(2\pi)^d}\,\frac{1}{p^2(p^2-M_V^2)}\,. \tag{3.10}
$$

The two correlators $\langle VVQ_AQ_A\rangle_1$ and $\langle VVQ_VQ_V\rangle_1$ are then found to get no contributions proportional to G_VF_V.

Next, we consider $\langle AVQ_AQ_V\rangle_i$. One first finds a set of 7 diagrams which contribute both to $\langle AVQ_AQ_V\rangle_1$ and $\langle AVQ_AQ_V\rangle_4$. The three diagrams drawn below give zero contribution



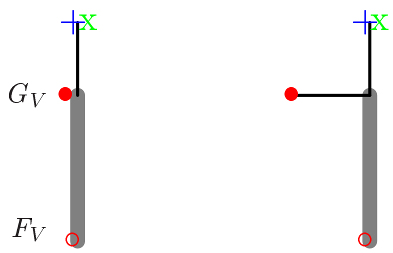
The next four diagrams are



Computing these four diagrams yields the following result for $\langle AVQ_AQ_V\rangle_1$

$$\langle AVQ_AQ_V\rangle_1 = G_V F_V \left(1-\frac{1}{d}\right)\int \frac{-i\, d^dp}{(2\pi)^d}\left(\frac{-d}{p^2(p^2-M_V^2)}+\frac{2}{(p^2-M_V^2)^2}\right), \tag{3.11}$$

these diagrams make an identical contribution also to $\langle AVQ_AQ_V\rangle_4$. In order to get the full contribution to $\langle AVQ_AQ_V\rangle_4$ we must also add the following two diagrams

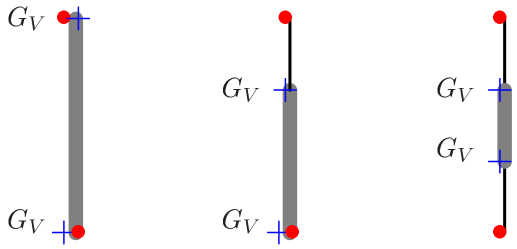


These diagrams are slightly more complicated to compute because one must allow for a small momentum to flow through the v_β source and expand the result in powers of k^2. One finds that the k^2 pole vanishes and then adding the constant contribution to that arising from the preceding four diagrams one finds that they cancel. In summary, the contributions proportional to $G_V F_V$ which are non vanishing read

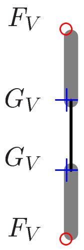
$$\begin{aligned}
\langle AAQ_AQ_A\rangle_1^{G_VF_V} &= \frac{3\,G_VF_V}{16\pi^2}\left(\frac{1}{6}+\log\frac{M_V^2}{\Lambda^2}\right)\\
\langle AAQ_VQ_V\rangle_1^{G_VF_V} &= -\frac{3\,G_VF_V}{16\pi^2}\left(\frac{1}{6}+\log\frac{M_V^2}{\Lambda^2}\right)\\
\langle AVQ_AQ_V\rangle_1^{G_VF_V} &= \frac{3}{2}\,\frac{G_VF_V}{16\pi^2}\left(\frac{1}{6}+\log\frac{M_V^2}{\Lambda^2}\right).
\end{aligned} \tag{3.12}$$

3.4 G_V^2 and $G_V^2 F_V^2$ contributions

These contributions are simple because a single correlator component is found to be non vanishing: $\langle AAQ_AQ_A\rangle_1$ in the case of G_V^2 and $\langle AAQ_VQ_V\rangle_1$ in the case of $G_V^2F_V^2$. There are three diagrams proportional to G_V^2



and a single one proportional to $G_V^2F_V^2$



After a small computation the following results are obtained,

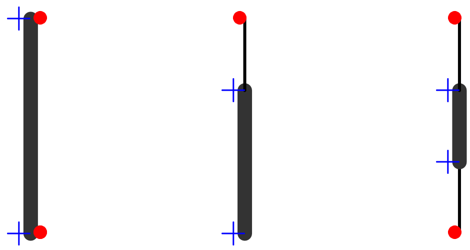
$$
\begin{aligned}
\langle AAQ_AQ_A\rangle_1 &= 4G_V^2\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\left(\frac{2-d}{M_V^2(p^2-M_V^2)}+\frac{d-1}{p^2(p^2-M_V^2)}\right)\\
\langle AAQ_VQ_V\rangle_1 &= -4\frac{G_V^2F_V^2}{F_0^2}\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\,\frac{1}{(p^2-M_V^2)^2}\,.
\end{aligned}
\tag{3.13}
$$

In summary, the non vanishing contributions proportional to G_V^2 read

$$
\begin{aligned}
\langle AAQ_AQ_A\rangle_1^{G_V^2} &= -\frac{3G_V^2}{16\pi^2}\left(\frac{1}{6}+\log\frac{M_V^2}{\Lambda^2}\right)\\
\langle AAQ_VQ_V\rangle_1^{G_V^2F_V^2} &= \frac{3G_V^2F_V^2}{16\pi^2F_0^2}\left(\frac{7}{6}+\log\frac{M_V^2}{\Lambda^2}\right).
\end{aligned}
\tag{3.14}
$$

3.5 Contributions proportional to c_d, $\tilde{c}_d$

The scalar singlet turns out to contribute only to $\langle AAQ_AQ_A\rangle_2$ and the scalar octet to $\langle AAQ_AQ_A\rangle_3$. The three Feynman diagrams to be computed are depicted below.



JHEP06(2004)047

The dependence on the photon gauge parameter ξ is found to cancel out, and the following results are obtained for the singlet and octet scalar contributions

$$\begin{aligned}\langle AAQ_AQ_A\rangle_2 &= 16\tilde{c}_d^2\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\,\frac{1}{p^2(M_{S_0}^2-p^2)}\\ \langle AAQ_AQ_A\rangle_3 &= 8c_d^2\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\,\frac{1}{p^2(M_S^2-p^2)}\,.\end{aligned} \tag{3.15}$$

As before, we have replaced the term $p_\alpha p_\beta$ in the integrands by $p^2 g_{\alpha\beta}/d$. There are three additional tadpole diagrams. Let us quote the result for completeness

$$\langle A_\alpha^a A_\beta^b Q_A^c Q_A^d\rangle^{tad} = i g_{\alpha\beta}\left(\frac{8c_d^2}{M_S^2}\,d^{sab}d^{scd}+\frac{16\tilde{c}_d^2}{M_{S_0}^2}\,\delta^{ab}\delta^{cd}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\,\frac{3}{p^2}\,. \tag{3.16}$$

Computing the integrals we obtain the following results for the scalar resonance contributions which are non vanishing

$$\begin{aligned}\langle AAQ_AQ_A\rangle_2^{\rm scal} &= 12\frac{\tilde{c}_d^2}{16\pi^2}\left(\log\frac{M_{S_0}^2}{\Lambda^2}+\frac{1}{6}\right)\\ \langle AAQ_AQ_A\rangle_3^{\rm scal} &= 6\frac{c_d^2}{16\pi^2}\left(\log\frac{M_S^2}{\Lambda^2}+\frac{1}{6}\right)\,.\end{aligned} \tag{3.17}$$

One notices that no dependence upon the gauge parameter ξ appears except in some of the terms proportional to F_A^2 and F_V^2. This is to be expected because, quite generally, the part proportional to ξ in the integral representation can be simplified by making use of chiral Ward identities and expressed in terms of 2-point Green's functions [31].

3.6 Comparison with the results of Baur and Urech

Up to this point we have used exactly the same resonance lagrangian terms as Baur and Urech [29] and we are therefore in a position to check their results for the chiral parameters K_i^r. Making use of the relations (2.14)–(2.18), for each resonance term contribution to the correlators (2.3) (2.4) we can solve for the K_i^r's which will then depend on the two scales μ and Λ. In ref. [29] the values of the two scales have been taken to be equal. Following this convention for the sake of comparison, and using the results derived above, we reproduce the results of ref. [29] for the parameters $K_1^r,\ldots,K_6^r$.

We disagree, however, on a number of contributions which concern K_{12}^r, K_{13}^r, K_{14}^r. Contrary to BU, we find that the following quantities are vanishing.

$$K_{13}^{G_VF_V} = K_{14}^{G_VF_V} = K_{12}^{G_V^2} = K_{13}^{G_V^2} = K_{14}^{G_V^2} = K_{12}^{G_V^2F_V^2} = K_{13}^{G_V^2F_V^2} = K_{14}^{G_V^2F_V^2} = 0\,. \tag{3.18}$$

In the case of K_{13}^r, K_{14}^r this follows from [31] where it is shown that they can be obtained from 2-point correlators $\langle Q_VQ_V\rangle$ and $\langle Q_AQ_A\rangle$. Obviously, these can only pick up contributions proportional to F_A^2 and F_V^2, which is indeed what we recover here. Similarly, one can also obtain K_{12}^r from a correlator with three currents $\langle PQ_AQ_V\rangle$ which picks up no contribution proportional to G_V^2.

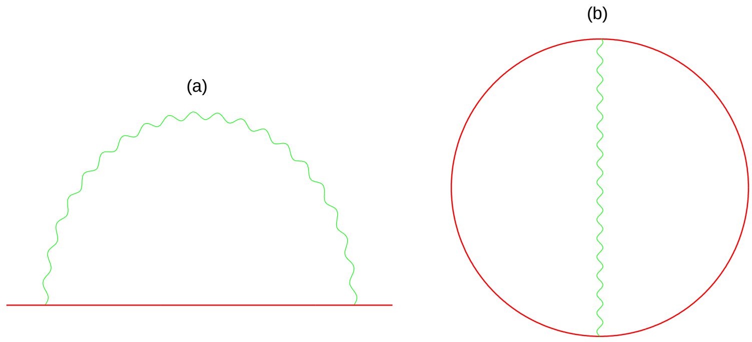


Figure 1: Typical divergent diagrams quadratic in the spurion sources (a) with an open quark line (b) with a closed quark line.

4. Short distance constraints

We now need to determine the divergence structure in the integral representation of eq. (2.5) according to QCD. One way to proceed is to determine the necessary counterterms which are generated in QCD with a source term as in eq. (2.1), where F_μ is a dynamical (massive) photon. This QED-like sector differs from ordinary QED by the fact that we have replaced the charge matrix Q by a vector (q_V) as well as an axial-vector (q_A) spurion source. We must consider this theory as a kind of effective theory and we assume that the divergences can be absorbed into a set of local lagrangian terms. We will restrict ourselves to an expansion quadratic in the spurion fields. This represents an important simplification since, for instance, we will only need the free photon propagator and thereby avoid the problems of the photon propagator at higher order in the presence of axial couplings [39]. Another restriction is that we will only consider contributions which are of leading order in N_c. Dimensional regularization together with the naive γ_5 prescription will be used.

4.1 QCD+QED counterterms

Let us consider QCD in the presence of dynamical photons and a source term of the form of eq. (2.1). The effective action is expanded to quadratic order in the spurions q_V and q_A and we drop the contributions which are subleading in N_c. We can then split the divergent terms in the effective action into two categories, (a) one which collects the graphs with an open quark line and, (b) one which collects the graphs with a closed quark loop (see figure 1). The former ones are those which will prove to be of relevance to us.

Let us first consider the contributions in category (b). The graphs in this category, at any order in α_s (in QCD the three gluon vertex has dimension p, the four gluon vertex has dimension zero), have dimensionality p^4. The corresponding local counterterms therefore must involve four derivatives. Examples of such terms are,

$$\langle \nabla^2 q_L \nabla^2 q_L + \nabla^2 q_R \nabla^2 q_R \rangle\,, \qquad G_{\mu\nu}G^{\mu\nu}\langle \nabla_\rho q_L \nabla^\rho q_L + \nabla_\rho q_R \nabla^\rho q_R \rangle\,, \qquad \text{etc}\ldots \tag{4.1}$$

If we restrict ourselves to the correlator combinations Π_i, eq. (2.19) (from which the contribution proportional to K_{14} drops out) then all the contributions from such counterterms also drop out or are suppressed in the large N_c counting.

We are finally left with the contributions from the first category. Taking into account the structure of QCD, all graphs in this category have dimension p, implying that the counterterms must involve one derivative. Furthermore, they must involve two spurions of the same kind (either q_L or q_R). One can form only two such independent terms

$$\mathcal{L} = \frac{i}{2} Z_2 \left\{ \bar{\psi}_L [q_L, \nabla^\mu q_L] \gamma_\mu \psi_L + \bar{\psi}_R [q_R, \nabla^\mu q_R] \gamma_\mu \psi_R \right\} + \\ + \frac{i}{2} Z_2' \left\{ \bar{\psi}_L q_L^2 \gamma_\mu D^\mu \psi_L + \bar{\psi}_R q_R^2 \gamma_\mu D^\mu \psi_R + \text{h.c.} \right\} \tag{4.2}$$

where

$$iD^\mu \psi_L = (i\partial^\mu + l_\mu + G_\mu)\psi_L \,, \qquad iD^\mu \psi_R = (i\partial^\mu + r_\mu + G_\mu)\psi_R \,. \tag{4.3}$$

At order zero in the strong coupling constant one has the well known QED result

$$Z_2 = Z_2' = \frac{\mu_0^{-2\epsilon}}{16\pi^2}(-\xi)[\Gamma(\epsilon) + \log(4\pi)] + O(\alpha_s) \,. \tag{4.4}$$

Equality between Z_2 and Z_2' need not hold beyond the lowest order.

Let us now discuss the contributions proportional to Z_2 and Z_2' to our basic correlators (2.3) (2.4). The first result is that the contribution proportional to Z_2' vanishes at leading order in N_c. This can be shown in a pedestrian way. Because of the conservation of chirality in one fermion loop graphs, the contributions concern only the correlators $\langle LLQ_LQ_L \rangle$ and $\langle RRQ_RQ_R \rangle$, it suffices to consider the former. For a given gluon field configuration $G_\mu(x)$ we can express the functional derivatives with respect to the sources in terms of exact fermion propagators $S_G(x,y)$ and we obtain the following expression

$$\begin{aligned} \langle L^a_\alpha L^b_\beta Q^c_L Q^d_L \rangle_G = \frac{Z_2'}{32} \int d^4x d^4y \Big[& - \delta(y) \langle \lambda^b \lambda^a \{\lambda^c, \lambda^d\} \rangle \langle \gamma^\alpha P_L S_G(x,0) \gamma^\beta P_L S_G(0,x) \rangle - \\ & - \delta(x) \langle \lambda^a \lambda^b \{\lambda^c, \lambda^d\} \rangle \langle \gamma^\beta P_L S_G(y,0) \gamma^\alpha P_L S_G(0,y) \rangle \\ & + \langle \lambda^a \lambda^b \{\lambda^c, \lambda^d\} \rangle \langle \gamma^\alpha P_L S_G(x,y) \gamma^\beta P_L S_G(y,0) \times \\ & \qquad \times [i\vec{\not\partial} + \not{G}] P_L S_G(0,x) \rangle + \\ & + \langle \lambda^b \lambda^a \{\lambda^c, \lambda^d\} \rangle \langle \gamma^\beta P_L S_G(y,x) \gamma^\alpha P_L S_G(x,0) \times \\ & \qquad \times [i\vec{\not\partial} + \not{G}] P_L S_G(0,y) \rangle \Big] \,, \end{aligned} \tag{4.5}$$

where P_L is the projector $P_L = 1/2(1-\gamma_5)$ and the notation $\vec{\not\partial}$ means that the derivatives are acting on the right. There is another similar contribution coming from the hermitian conjugate part of the lagrangian. Using the equation obeyed by the Dirac propagator

$$[i\vec{\not\partial} + \not{G}] P_L S_G(x,y) = P_R \delta^4(x-y) \,, \tag{4.6}$$

we indeed find that the various terms in eq. (4.5) cancel each other. We have thus shown that the only relevant contributions from the counterterms are those proportional to Z_2. The calculation of these is performed by computing the functional derivatives according to the definition of eq. (2.3) and using the following result for the two-point correlators that appear

$$\int d^4x e^{ikx} \langle A^s_\alpha(x) A^t_\beta(0) \rangle = iF_0^2 \delta^{st} \left(\frac{k_\alpha k_\beta}{k^2} - g_{\alpha\beta} \right) , \qquad \int d^4x \langle V^s_\alpha(x) V^t_\beta(0) \rangle = 0 \,. \tag{4.7}$$

We find that the counterterms contribute to the following components of our basic correlators

$$\begin{aligned}
\langle AAQ_AQ_A\rangle_1^{ct} &= -Z_2F_0^2 \\
\langle AAQ_VQ_V\rangle_1^{ct} &= -Z_2F_0^2 \\
\langle AVQ_AQ_V\rangle_1^{ct} &= -\frac{1}{2}Z_2F_0^2 \\
\langle AVQ_AQ_V\rangle_4^{ct} &= -\frac{1}{2}Z_2F_0^2 \,.
\end{aligned} \tag{4.8}$$

There are no contributions to the other components.

4.2 Summary of the $\log \Lambda^2$ dependence

Let us consider the combinations of correlators Π_i introduced in eqs. (2.19) (2.23). In all these combinations the contribution from the coupling K_{14}^r drops out. In the combinations $\Pi_1, \ldots, \Pi_6$ as well as Π_7, Π_9, the contributions from the renormalization parameter Z_2 also drops out, as is seen from using eq. (4.8). As a consequence, these quantities must be finite. More precisely, finiteness holds in the sense of dimensional regularization which was used to establish the form of the counterterms in the preceding section: one must compute the photon loop integral in d dimensions and then take the limit $d \to 4$ which must be finite. From these eight combinations one can then determine the chiral couplings $K_1^r, \ldots, K_6^r$ as well as K_{13}^r and C. The situation is different for Π_8 since, in that case, a contribution proportional to Z_2 is present. The integral is not finite in general. It is finite to zeroth order in the strong coupling α_s when the gauge parameter $\xi = 0$. This limitation affects the determination of the coupling K_{12}^r.

Let us examine how these finiteness conditions are satisfied when the relevant correlators are computed from the minimal resonance lagrangian of eqs. (3.2), (3.3). Collecting the results presented in detail in section 3, we obtain the following dependence upon the regularization scale Λ

$$\begin{aligned}
\Pi_1 &\sim -\frac{3}{2}\frac{\log\Lambda^2}{16\pi^2}[F_0^2 + F_A^2 - 2G_V^2] \\
\Pi_2 &\sim -\frac{3}{2}\frac{\log\Lambda^2}{16\pi^2}\left[F_0^2 + F_V^2 - 4G_VF_V + 2G_V^2\frac{F_V^2}{F_0^2}\right] \\
\Pi_3 &\sim -6\frac{\log\Lambda^2}{16\pi^2}\,[c_d^2] \\
\Pi_4 &= 0 \\
\Pi_5 &\sim -4\frac{\log\Lambda^2}{16\pi^2}\,[3\tilde{c}_d^2 - c_d^2] \\
\Pi_6 &= 0 \\
\Pi_7 &\sim \frac{3}{4}\frac{\log\Lambda^2}{16\pi^2}[F_V^2 - F_0^2 - F_A^2] \\
\Pi_8 &\sim -\frac{3}{8}\frac{\log\Lambda^2}{16\pi^2}[F_V^2 - F_0^2 - F_A^2 + 4(G_VF_V - F_V^2)] \\
\Pi_9 &\sim 3\frac{\log\Lambda^2}{16\pi^2}[M_V^2F_V^2 - M_A^2F_A^2]\,.
\end{aligned} \tag{4.9}$$

At this level, there is at first one consistency condition which is satisfied, but trivially so, since Π_4 is identically vanishing. In the sequel (section 5.6) we will consider physically relevant interactions and attempt to satisfy the condition in a less trivial way. The consistency conditions for Π_7 and Π_9 can be satisfied and they generate the two Weinberg sum rule relations

$$F_V^2 = F_0^2 + F_A^2 , \qquad M_V^2 F_V^2 = M_A^2 F_A^2 . \tag{4.10}$$

These relations will be assumed to hold from now on. Furthermore, we will assume the following relation to hold,

$$G_V F_V = F_0^2 \tag{4.11}$$

which follows [35] from imposing that the asymptotic behaviour of the vector form-factor of the pion

$$\lim_{t\to\infty} t F_\pi^V(t) = 0 \tag{4.12}$$

is satisfied. Using this, we observe that there are only two consistency conditions which are fulfilled, which are those associated with Π_7 and Π_9. The corresponding evaluation of Π_7 and Π_9 gives

$$\begin{aligned}\Pi_7 &= \frac{3}{4}\frac{F_0^2}{16\pi^2}\left[\log\frac{M_V^2}{M_\gamma^2} + 2 - \frac{\log z}{z-1}\right] \\ \Pi_9 &= 3M_V^2 \frac{F_0^2}{16\pi^2}\frac{\log z}{z-1} ,\end{aligned} \tag{4.13}$$

where z denotes the mass ratio

$$z = \frac{M_A^2}{M_V^2} \tag{4.14}$$

which one expects to be approximately equal to 2. We expect Π_5 and Π_6 to vanish as a result of the leading large N_c approximation. This condition is trivially satisfied for Π_6 and it can be imposed for Π_5 by setting $3\tilde{c}_d^2 = c_d^2$.

5. Beyond the $O(p^4)$ resonance lagrangian

5.1 Survey of $O(p^6)$ terms

In order to comply with the other consistency conditions, it is necessary to enlarge the resonance lagrangian of eq. (3.1). Since this lagrangian (for a given resonance content) contains all terms of order p^4, it is natural to investigate terms of order six. Generically, let us designate a resonance building block by R (which counts as p^2), a chiral building block by π (which counts as p) and a source term by J (which counts as p^2). In this schematic notation, there are four classes of terms of order p^4 which involve at least one resonance

$$RR,\ RJ,\ R\pi,\ R\pi\pi . \tag{5.1}$$

At order p^6 one can have the same terms but involving two more derivatives and, in addition, one can have terms formed from three, four as well as five building blocks

$$RRR,\ RR\pi,\ RRJ,\ RJJ,\ RJ\pi,\ RR\pi\pi,\ RJ\pi\pi,\ R\pi\pi\pi,\ R\pi\pi\pi\pi . \tag{5.2}$$

We will not attempt to classify and discuss all possible remaining terms in this work. We will simply consider (except in section 5.6) sets of terms of the form $RR\pi$. The first set will be labeled as $a_1\rho\pi$ and has the following form

$$\begin{aligned}\mathcal{L}_{a_1\rho\pi} &= i\left(G_1 g^{\mu_0\mu_1}g^{\mu_2\mu_3} + G_2 g^{\mu_0\mu_2}g^{\mu_3\mu_1} + G_3 g^{\mu_0\mu_3}g^{\mu_1\mu_2}\right)\langle\,[\nabla_{\mu_0}V_{\mu_1\lambda}, A_{\mu_2}{}^{\lambda}]u_{\mu_3}\rangle + \\ &+ i\left(H_1 g^{\mu_0\mu_1}g^{\mu_2\mu_3} + H_2 g^{\mu_0\mu_2}g^{\mu_3\mu_1} + H_3 g^{\mu_0\mu_3}g^{\mu_1\mu_2}\right)\langle\,[V_{\mu_1\lambda}, A_{\mu_2}{}^{\lambda}]\nabla_{\mu_0}u_{\mu_3}\rangle\,. \end{aligned} \tag{5.3}$$

We will first show that by adding these terms to the minimal resonance lagrangian we can reproduce the $\langle VAP\rangle$ 3-point function constructed in ref. [31].[1] In addition, we will then show that it is possible to satisfy the consistency conditions associated with Π_1 and Π_2, and these provides a number of Weinberg-type sum rule relations. In an analogous way, we introduce an $a_0f_1\pi$ coupling (we assume nonet symmetry which reduces the number of couplings),

$$\mathcal{L}_{a_0f_1\pi} = c_A\langle S\{\nabla^\mu A_{\mu\nu}, u^\nu\}\rangle \tag{5.4}$$

which will allow us to satisfy the consistency condition on Π_3.

Finally, we will consider a set of terms of the type $\omega\rho\pi$ which contribute to Π_4. A calculation which is equivalent to an evaluation of this contribution was performed in ref. [7] using the vector, instead of the anti-symmetric tensor, formulation for the vector fields. A classification of the terms in this latter formulation was recently presented by Ruiz-Femenia et al. [41], we list below those of relevance to us,

$$\begin{aligned}\mathcal{L}_{\omega\rho\pi} = \epsilon_{\mu\nu\rho\sigma}\Big\{ & d_3\langle\{\nabla_\lambda V^{\mu\nu}, V^{\rho\lambda}\}u^\sigma\rangle + d_4\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\lambda}\}u_\lambda\rangle + \\ & + \frac{c_5}{M_V}\langle\{\nabla_\lambda V^{\mu\nu}, f_+^{\rho\lambda}\}u^\sigma\rangle + \frac{c_6}{M_V}\langle\{\nabla_\lambda V^{\mu\lambda}, f_+^{\rho\sigma}\}u^\nu\rangle + \\ & + \frac{c_7}{M_V}\langle\{\nabla^\sigma V^{\mu\nu}, f_+^{\rho\lambda}\}u_\lambda\rangle\Big\}\,. \end{aligned} \tag{5.5}$$

It enabled them to reproduce the 3-point function $\langle VVP\rangle$ as constructed in ref. [42]. The consistency condition on Π_4 is somewhat more complicated to satisfy in the antisymmetric tensor than in the vector formalism. This will be explained in more detail below.

5.2 $\mathcal{L}_{a_1\rho\pi}$ constraints from $\langle VAP\rangle$

Let us examine the constraints on the six coupling constants of the $a_1\rho\pi$ lagrangian (5.3) which arise from the 3-point correlator $\langle VAP\rangle$. This correlator is defined as follows

$$W_{\alpha\beta}(p,q) = \int d^4x d^4y e^{ipx+iqy}\langle 0|T\,(V^1_\alpha(x)A^2_\beta(y)P^3(0))|0\rangle\,. \tag{5.6}$$

Assuming an ansatz in terms of rational functions with a minimal number of resonance poles (i.e. one pion, one vector and one axial-vector resonance poles) it can be shown [31]

[1] An extension was recently considered [40] which includes a π' resonance multiplet and a corresponding resonance chiral lagrangian was constructed. The result for the coupling constant combination $G_1+G_2+2G_3$ which is needed in the following turns out to be unaffected by this extension.

JHEP06(2004)047

that this correlator is uniquely determined from (a) the chiral Ward identities, (b) the constraint to match the leading asymptotic scaling behaviour,

$$\lim_{\lambda\to\infty} W_{\alpha\beta}(\lambda p,\lambda q) = \frac{\langle \bar{u}u\rangle}{\lambda^2}\left\{\frac{(p_\alpha+2q_\alpha)q_\beta - q^2 g_{\alpha\beta}}{q^2 l^2} + \frac{(p^2-q^2-l^2)P_{\alpha\beta}}{2p^2q^2l^2} - \frac{Q_{\alpha\beta}}{p^2q^2l^2}\right\} \tag{5.7}$$

with $l^2=(p+q)^2$,

$$\begin{aligned} P_{\alpha\beta} &= p_\beta q_\alpha - p.q g_{\alpha\beta} \\ Q_{\alpha\beta} &= p^2 q_\alpha q_\beta + q^2 p_\alpha p_\beta - p.q p_\alpha q_\beta - p^2 q^2 g_{\alpha\beta}\,, \end{aligned} \tag{5.8}$$

and, finally, (c) the asymptotic constraint on the vector form factor of the pion eq. (4.12). More detailed analysis of asymptotic constraints related to this 3-point correlator were performed in ref. [43]. The result, for $\langle VAP\rangle$, reads

$$\begin{aligned} W_{\alpha\beta}(p,q) \;=\; \langle\bar{u}u\rangle\Bigg\{ &\frac{(p_\alpha+2q_\alpha)q_\beta - q^2 g_{\alpha\beta}}{q^2l^2} + \\ &+\frac{F(p^2,q^2,l^2)\,P_{\alpha\beta}}{(p^2-M_V^2)(q^2-M_A^2)\,l^2} + \frac{G(p^2,q^2,l^2)\,Q_{\alpha\beta}}{(p^2-M_V^2)(q^2-M_A^2)q^2l^2}\Bigg\} \end{aligned} \tag{5.9}$$

with

$$\begin{aligned} F(p^2,q^2,l^2) &= \frac{1}{2}(p^2-q^2-l^2) + M_V^2 - M_A^2 \\ G(p^2,q^2,l^2) &= -q^2 + 2M_A^2\,. \end{aligned} \tag{5.10}$$

Let us compute the VAP Green's function from the resonance lagrangian $\mathcal{L}_{\rm res}^{(4)} + \mathcal{L}_{a_1\rho\pi}$. We find, for the F and G polynomials,

$$\begin{aligned} F(p^2,q^2,l^2) &= p^2\left[-\frac{F_A^2}{F_0^2} - \frac{F_VF_A}{F_0^2}(-2G_1-G_2-2G_3+H_1+H_2)\right] + \\ &\quad +q^2\left[\frac{F_V^2-2G_VF_V}{F_0^2} - \frac{F_VF_A}{F_0^2}(G_2+2G_3+H_1+H_2)\right] + \\ &\quad +l^2\left(\frac{-F_VF_A}{F_0^2}\right)[-G_2-2G_3+H_1+H_2+4H_3] + \\ &\quad +\frac{1}{F_0^2}\left[M_V^2F_A^2 - M_A^2(F_V^2-2G_VF_V)\right] \\ G(p^2,q^2,l^2) &= q^2\left[-\frac{2G_VF_V}{F_0^2} - \frac{2F_VF_A}{F_0^2}(-G_1+H_1+H_2)\right] + 2M_A^2\frac{G_VF_V}{F_0^2}\,. \end{aligned} \tag{5.11}$$

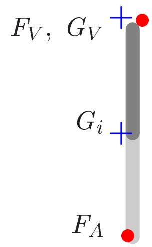
Let us first compare the constant terms in eqs. (5.11) and (5.10). Provided that the two Weinberg relations eq. (4.10) are satisfied together with the relation $G_VF_V=F_0^2$, one finds that the constant terms are in exact agreement. Next, the following three independent relations between the coupling constants G_i, H_i

$$\begin{aligned} G_1+G_2+2G_3 &= \frac{F_A}{F_V} \\ H_1+H_2 &= \frac{F_A}{2F_V} - \frac{F_V}{2F_A} + G_1 \\ 2H_3 &= \frac{F_V}{2F_A} - G_1 \end{aligned} \tag{5.12}$$

are found to be the necessary and sufficient conditions for the polynomials in eqs. (5.11) and (5.10) to be completely identical.

5.3 Linear contributions from $\mathcal{L}_{a_1\rho\pi}$

Let us now return to the 4-point correlators and consider the contributions which are linear in the couplings of eq. (5.3). The couplings H_i do not contribute because derivatives of the sources are involved. There are contributions to $\langle AAQ_AQ_A\rangle_1$, $\langle AAQ_VQ_V\rangle_1$ and $\langle AVQ_AQ_V\rangle_1$ which we consider in turn. There is one kind of diagram relevant for $\langle AAQ_AQ_A\rangle_1$



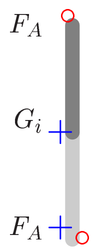
The upper vertex involves either F_V or G_V. These diagrams give

$$\langle AAQ_AQ_A\rangle_1 = -4F_A(F_V-2G_V)\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\left[\frac{(d-2)G_2}{M_V^2(M_A^2-p^2)}+\right. \\ \left. +\frac{G_{123}}{(M_A^2-p^2)(M_V^2-p^2)}\right] \tag{5.13}$$

where

$$G_{123}\equiv G_1+G_2+2G_3\,, \tag{5.14}$$

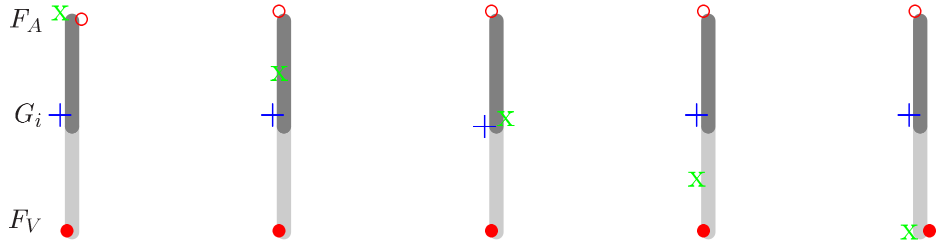
denotes the combination which was determined from $\langle VAP\rangle$. In the case of the correlator $\langle AAQ_VQ_V\rangle_1$, there is again one kind of diagram which makes a linear contribution



and the result is very similar to the preceding one

$$\langle AAQ_VQ_V\rangle_1 = -4F_V^2\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\left[\frac{(d-2)G_1}{M_A^2(M_V^2-p^2)}+\frac{G_{123}}{(M_A^2-p^2)(M_V^2-p^2)}\right]. \tag{5.15}$$

Next, in order to obtain $\langle AVQ_AQ_V\rangle$ one must calculate a set of five diagrams which are depicted below,



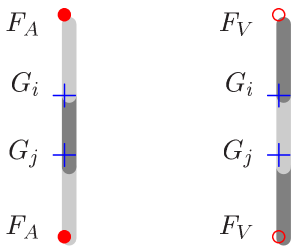
Computing these diagrams gives

$$
\begin{aligned}
\langle AVQ_AQ_V\rangle_1 = -F_AF_VG_{123}\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\Bigg[&\frac{d-2}{(M_A^2-p^2)(M_V^2-p^2)}+\\
&+\frac{2M_A^2}{(M_A^2-p^2)^2(M_V^2-p^2)}+\\
&+\frac{2M_V^2}{(M_A^2-p^2)(M_V^2-p^2)^2}\Bigg]
\end{aligned}
\tag{5.16}
$$

which is seen to be proportional to the coupling constant combination G_{123}.

5.4 Quadratic contributions from $\mathcal{L}_{a_1\rho\pi}$ and consistency conditions for Π_1, Π_2, Π_8

We turn now to the contributions which are quadratic in the couplings G_i. These concern only the two correlators $\langle AAQ_AQ_A\rangle_1$, $\langle AAQ_VQ_V\rangle_1$ and they are given respectively from the two kinds of diagrams depicted below



The computation gives the following result for $\langle AAQ_AQ_A\rangle_1$,

$$
\langle AAQ_AQ_A\rangle_1 = 4F_A^2\left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\left\{\frac{(G_{123})^2\,p^2}{(M_A^2-p^2)^2(M_V^2-p^2)}+\frac{G_2^2\,(2-d)\,p^2}{M_V^2(M_A^2-p^2)^2}\right\}.
\tag{5.17}
$$

The result for $\langle AAQ_VQ_V\rangle_1$ is obtained simply by replacing F_A by F_V and by interchanging M_A, M_V and G_1, G_2 in the expression above.

We can now collect all the contributions which involve the coupling constants G_i,

$$
\begin{aligned}
\langle AAQ_AQ_A\rangle_1^{G_i} = \frac{3F_A(F_V-2G_V)}{16\pi^2}\Bigg\{&-2G_2\,z\left(\frac{7}{6}+\log\frac{M_A^2}{\Lambda^2}\right)+\\
&+G_{123}\left[\frac{1}{6}+\frac{z\log z}{z-1}+\log\frac{M_V^2}{\Lambda^2}\right]\Bigg\}+
\end{aligned}
$$

$$+\frac{3F_A^2}{16\pi^2}\left\{4G_2^2\,z\left(\frac{5}{3}+\log\frac{M_A^2}{\Lambda^2}\right)-\right.$$

$$\left.-\,(G_{123})^2\left[\frac{7z-1}{6(z-1)}-\frac{\log z}{(z-1)^2}+\log\frac{M_A^2}{\Lambda^2}\right]\right\} \tag{5.18}$$

$$\langle AAQ_VQ_V\rangle_1^{G_i} = \frac{3F_AF_V}{16\pi^2}\left\{\frac{-2\,G_1}{z}\left(\frac{7}{6}+\log\frac{M_V^2}{\Lambda^2}\right)+\right.$$

$$\left.+\,G_{123}\left[\frac{1}{6}+\frac{z\log z}{z-1}+\log\frac{M_V^2}{\Lambda^2}\right]\right\}+$$

$$+\frac{3F_V^2}{16\pi^2}\left\{4\frac{G_1^2}{z}\left(\frac{5}{3}+\log\frac{M_V^2}{\Lambda^2}\right)-\right.$$

$$\left.-\,(G_{123})^2\left[\frac{7-z}{6(1-z)}+\frac{z^2\log z}{(z-1)^2}+\log\frac{M_V^2}{\Lambda^2}\right]\right\}, \tag{5.19}$$

$$\langle AVQ_AQ_V\rangle_1^{G_i} = \frac{3}{2}\frac{F_AF_VG_{123}}{16\pi^2}\left[\frac{1}{6}+\frac{z\log z}{z-1}+\log\frac{M_V^2}{\Lambda^2}\right]. \tag{5.20}$$

Adding these contributions to those computed in section 3 it is now possible to satisfy the consistency conditions associated with Π_1, Π_2 and Π_8. Firstly one observes that the Λ scale dependence drops out from the combination Π_8 automatically provided the first relation (5.12) holds for G_{123}. Requiring that the consistency conditions for Π_1 and Π_2 be satisfied yields two Weinberg-type relations for G_1 and G_2

$$\begin{aligned} 8G_1^2-\frac{4}{\sqrt{z}}G_1-1 &= 0\\ 8G_2^2-\frac{4}{\sqrt{z}}(2-z)G_2-1 &= 0\,. \end{aligned} \tag{5.21}$$

The previous determinations of F_V, F_A, G_V and of the combination $G_1+G_2+2G_3$ have been used. Since these equations are quadratic, one could be concerned about the existence of real solutions. Real solutions do exist for any value of the mass ratio z and, a priori, one has a set of four different solutions. By studying the behaviour as a function of z we can reduce the multiplicity. We note that the Weinberg equations (4.10) have real solutions provided z lies in the range $1\le z<\infty$. Firstly, let $z\to 1$: this corresponds to a situation where chiral symmetry gets restored and one must have $G_1=G_2$ in this case. This argument reduces the multiplicity to just two solutions, which we can write,

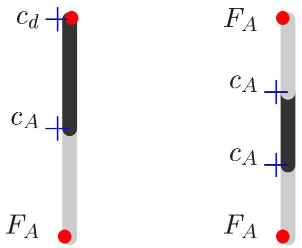
$$\begin{aligned} G_1 &= \frac{1}{4\sqrt{z}}(1+\sigma\sqrt{1+2z})\\ G_2 &= \frac{1}{4\sqrt{z}}(2-z+\sigma\sqrt{(2-z)^2+2z})\,, \qquad \sigma=\pm 1\,. \end{aligned} \tag{5.22}$$

We can further reduce the multiplicity by observing the behaviour as $z\to\infty$. In this limit, G_1 remains finite while in the case of G_2 only one solution remains finite corresponding to $\sigma=1$. Correspondingly, the divergence in Π_1 is logarithmic for $\sigma=1$ while it is linear for $\sigma = -1$. The solution with $\sigma = 1$, therefore, appears to be the most plausible choice. Using this solution, the following determinations are obtained for Π_1, Π_2 and Π_8

$$\begin{aligned}
\Pi_1 &= \frac{3}{2}\frac{F_0^2}{16\pi^2}\Bigg\{\log\frac{M_\gamma^2}{M_V^2} + \frac{(z^3-5z^2+5z+1)\log z}{(z-1)^3} + \\
&\qquad + \frac{z^3-2z^2+7z-10}{2(z-1)^2} - \frac{(z-2)\sqrt{z^2-2z+4}}{2(z-1)}\Bigg\} \\
\Pi_2 &= \frac{3}{2}\frac{F_0^2}{16\pi^2}\Bigg\{\log\frac{M_\gamma^2}{M_V^2} - \frac{2z^2\log z}{(z-1)^3} + \\
&\qquad + \frac{6z^3-7z^2+6z-1}{2z(z-1)^2} + \frac{\sqrt{2z+1}}{2z(z-1)}\Bigg\} \\
\Pi_8 &= \frac{3}{8}\frac{F_0^2}{16\pi^2}\left(-\log\frac{M_\gamma^2}{M_V^2} + \frac{(3z+1)\log z}{(z-1)^2} - \frac{4z}{z-1}\right).
\end{aligned} \tag{5.23}$$

5.5 Contributions from $\mathcal{L}_{a_0 f_1 \pi}$ and the consistency condition for Π_3

By analogy with the preceding we now consider couplings which involve the scalar mesons of the type $a_0 f_1 \pi$ which will contribute to $\langle AAQ_AQ_A\rangle_{2,3}$. There is only one lagrangian term which matters, shown in eq. (5.4) (assuming nonet symmetry). The term in which the derivative acts on u_ν does not contribute to our correlators nor to the physical decay amplitude which is discussed below. There are two kinds of diagrams which contribute to the relevant correlators



Computing the diagrams gives

$$\begin{aligned}
\langle AAQ_AQ_A\rangle_3 &= \left(1-\frac{1}{d}\right)\int\frac{-i\,d^dp}{(2\pi)^d}\left\{\frac{8\sqrt{2}c_dc_AF_A}{(M_A^2-p^2)(M_S^2-p^2)} + \frac{4c_A^2F_A^2p^2}{(M_A^2-p^2)^2(M_S^2-p^2)}\right\} \\
&= \frac{-6\sqrt{2}c_Ac_dF_A}{16\pi^2}\left(\frac{1}{6} + \log\frac{M_S^2}{\Lambda^2} + \frac{M_A^2}{M_A^2-M_S^2}\log\frac{M_A^2}{M_S^2}\right) + \\
&\quad + \frac{3c_A^2F_A^2}{16\pi^2}\left(\frac{1}{6} + \log\frac{M_S^2}{\Lambda^2} + \frac{M_A^2}{M_A^2-M_S^2} + \frac{M_A^2(M_A^2-2M_S^2)}{(M_A^2-M_S^2)^2}\log\frac{M_A^2}{M_S^2}\right).
\end{aligned} \tag{5.24}$$

We can now add this result to the contribution (3.17) proportional to c_d^2, and the consistency condition translates into a simple relation between the coupling constants c_d and c_a

$$c_AF_A = \sqrt{2}c_d\,. \tag{5.25}$$

This allows one to obtain a consistent determination of the correlator component $\langle AAQ_AQ_A\rangle_3$ associated with the scalar mesons,

$$\Pi_3 = \langle AAQ_AQ_A\rangle_3 = \frac{6c_d^2 M_A^2}{16\pi^2(M_A^2 - M_S^2)}\left(1 - \frac{M_A^2}{M_A^2 - M_S^2}\log\frac{M_A^2}{M_S^2}\right). \tag{5.26}$$

We can compare the above determination of the coupling c_A with experiment. Let us consider the decay process $f_1(1285) \to a_0(980) + \pi$. Using the lagrangian above (5.4) the expression for the decay width is easily derived,

$$\Gamma_{f_1\to a_0\pi} = c_A^2 \frac{p_{cm}\,(M_A^2 - M_S^2)^2}{8\pi M_{f_1}^2 F_0^2}\,. \tag{5.27}$$

Using the value $c_d \simeq 32\,\mathrm{MeV}$ from ref. [35], $M_A = \sqrt{2}M_\rho$ (and physical masses in kinematical factors) one obtains

$$\Gamma_{f_1\to a_0\pi} \simeq 7.7\,\mathrm{MeV}\,. \tag{5.28}$$

This result is in rather good agreement with the experimental one $\Gamma^{\mathrm{exp}}_{f_1\to a_0\pi} = 8.6\pm 1.7\,\mathrm{MeV}$ from ref. [44].

5.6 Contributions from $\mathcal{L}_{\omega\rho\pi}$ in the vector vs. tensor formalism and Π_4

We discuss here contributions from $\mathcal{L}_{\omega\rho\pi}$ eq. (5.5) which are likely to be important and were also not accounted for in the minimal resonance lagrangian. One faces a technical difficulty that the lagrangian eq. (5.5) involves the Levi-Civita ϵ tensor and this makes continuation to d dimensional space ambiguous. One would therefore like to impose a stronger version of the consistency condition that the integral be finite in four dimensions. This condition is rather simple to enforce if one uses the vector formalism. Let us first briefly review what would be the result in this case and then we will show how to proceed in the case of the tensor formalism. An extensive study of resonance lagrangian terms in the vector formalism can be found in ref. [45]. In this formulation, the terms including the $\omega\rho\pi$ type couplings read (we use the same notation as [35]),

$$\begin{aligned}\mathcal{L}^V_{\omega\rho\pi} &= -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu} - 2m_V^2 V_\mu V^\mu\rangle - \frac{1}{2\sqrt{2}} f_V \langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \\ &\quad + \frac{g_1^V}{2\sqrt{2}}\,\epsilon^{\mu\nu\alpha\beta}\langle\{u_\mu, V_\nu\} f_{+\alpha\beta}\rangle + \frac{g_2^V}{2}\,\epsilon^{\mu\nu\alpha\beta}\langle\{u_\mu, V_\nu\} V_{\alpha\beta}\rangle\,,\end{aligned} \tag{5.29}$$

where

$$V_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu\,. \tag{5.30}$$

In this description the consideration of short distance conditions imposes that chiral $O(p^4)$ terms be added [35] but these do not play any role for our calculation. Computing the correlator $\langle AAQ_VQ_V\rangle_3$ from this lagrangian, one finds

$$\langle A_\alpha A_\beta Q_V Q_V\rangle_3 = 8\int \frac{d^4p}{(2\pi)^4}\frac{(p^2 g_{\alpha\beta} - p_\alpha p_\beta)\left[p^2(g_1^V + 2g_2^V f_V) - M_V^2 g_1^V\right]^2}{p^2(M_V^2 - p^2)^3}\,. \tag{5.31}$$

The integral in eq. (5.31) can be defined in four dimensions without difficulty, it suffices that the following relation holds among the coupling constants

$$g_1^V + 2g_2^V f_V = 0\,. \tag{5.32}$$

The result for the correlator then reads,

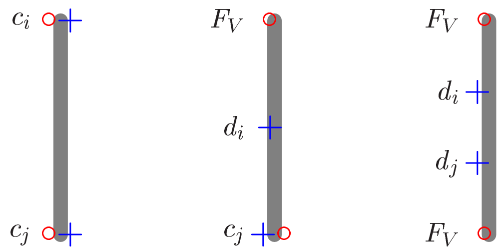
$$\langle AAQ_VQ_V\rangle_3^{\omega\rho\pi} = \frac{F_0^2}{16\pi^2}(3\tilde{g}_V^2)\,, \qquad \tilde{g}_V = \frac{g_1^V M_V}{F_0}\,. \tag{5.33}$$

The value of $\tilde{g}_V$ was estimated in ref. [7] from the $\omega \to \pi\gamma$ decay rate, $|\tilde{g}_V| = 0.91 \pm 0.03$. Under the assumption of nonet symmetry (which is rather well satisfied experimentally for the vectors), one also has

$$\langle AAQ_VQ_V\rangle_2^{\omega\rho\pi} = \frac{2}{3}\langle AAQ_VQ_V\rangle_3^{\omega\rho\pi}\,. \tag{5.34}$$

From eqs. (5.33), (5.34) one can deduce an estimate, which is identical to the one performed in ref. [7], for the combination of K_i's which is involved in the radiative corrections to the process $\pi^0 \to 2\gamma$.

Let us now reconsider the problem using the antisymmetric tensor formulation, i.e. starting from the lagrangian of eq. (5.5). One has three kinds of diagrams to compute which are shown below



Evaluating these various diagrams we obtain $\langle A_\alpha A_\beta Q_V Q_V\rangle_3$ in the form of the following four-dimensional integral

$$\begin{aligned}\langle A_\alpha A_\beta Q_V Q_V\rangle_3 = \int \frac{d^4p}{(2\pi)^4}\Bigg\{ &\frac{48\,p_\alpha p_\beta}{M_V^4}\left(\frac{M_V F_V(d_3+d_4)}{M_V^2-p^2} - \sqrt{2}(c_5+c_7)\right)^2 + \\ &+ \frac{32(p^2 g_{\alpha\beta} - p_\alpha p_\beta)}{M_V^2(M_V^2-p^2)}\left(\frac{2M_V F_V d_3}{M_V^2-p^2} - \sqrt{2}(c_5-c_6)\right)^2\Bigg\}\,. \end{aligned} \tag{5.35}$$

The integral in eq. (5.35) does not converge in four dimensions unless all the couplings c_i and d_i vanish identically. In order to remove the divergence, in this case, we introduce a more general set of resonance lagrangian terms, which include four building blocks

$$\begin{aligned}\mathcal{L}^{4bb} = &(x_1 g^{\mu_0\mu_1}g^{\mu_2\mu_3} + x_2 g^{\mu_0\mu_2}g^{\mu_1\mu_3} + x_3 g^{\mu_0\mu_3}g^{\mu_1\mu_2})\langle\{f^{+\lambda}{}_{\mu_0}, u_{\mu_1}\}\{f^+_{\lambda\mu_2}, u_{\mu_3}\}\rangle + \\ &+\sqrt{2}(y_1 g^{\mu_0\mu_1}g^{\mu_2\mu_3} + y_2 g^{\mu_0\mu_2}g^{\mu_1\mu_3} + y_3 g^{\mu_0\mu_3}g^{\mu_1\mu_2})\langle\{V^\lambda{}_{\mu_0}, u_{\mu_1}\}\{f^+_{\lambda\mu_2}, u_{\mu_3}\}\rangle + \\ &+(z_1 g^{\mu_0\mu_1}g^{\mu_2\mu_3} + z_2 g^{\mu_0\mu_2}g^{\mu_1\mu_3} + z_3 g^{\mu_0\mu_3}g^{\mu_1\mu_2})\langle\{V^\lambda{}_{\mu_0}, u_{\mu_1}\}\{V_{\lambda\mu_2}, u_{\mu_3}\}\rangle\,. \end{aligned} \tag{5.36}$$

These terms generate the following additional contribution to the correlator

$$\langle A_\alpha A_\beta Q_V Q_V\rangle_3^{4bb} = \int \frac{d^4p}{(2\pi)^4}\Bigg\{24p^2 g_{\alpha\beta}\left(\frac{2x_{123}}{p^2} - \frac{F_V y_{123}}{p^2(M_V^2-p^2)} + \frac{F_V^2 z_{123}}{p^2(M_V^2-p^2)^2}\right) - \\ - 16(p^2 g_{\alpha\beta} - p_\alpha p_\beta)\left(\frac{2x_{13}}{p^2} - \frac{F_V y_{13}}{p^2(M_V^2-p^2)} + \frac{F_V^2 z_{13}}{p^2(M_V^2-p^2)^2}\right)\Bigg\}. \tag{5.37}$$

In this expression we have used the notation

$$x_{123} = x_1 + 2x_2 + x_3\,, \qquad x_{13} = x_1 + x_3\,, \tag{5.38}$$

and similarly with y_i, z_i.

The convergence constraint in four dimension can now be satisfied: it imposes $c_5 + c_7 = 0$ and determines the values of the six combinations of parameters which appear in eq. (5.37) in terms of d_3, $d_3 + d_4$ and $c_5 - c_6$. Collecting all the pieces, one finds that all the couplings drop out except d_3 and the correlator $\langle A_\alpha A_\beta Q_V Q_V\rangle_3$ is then expressed as a convergent integral,

$$\langle A_\alpha A_\beta Q_V Q_V\rangle_3 = 128 M_V^2 F_V^2 d_3^2 \int \frac{d^4p}{(2\pi)^4}\frac{p^2 g_{\alpha\beta} - p_\alpha p_\beta}{p^2(M_V^2-p^2)^3}\,. \tag{5.39}$$

This expression is exactly analogous to the one obtained upon using the vector formalism and we write the final result in the same form,

$$\Pi_4 = \langle AAQ_V Q_V\rangle_3 = \frac{F_0^2}{16\pi^2}(3\tilde{g}_{AT}^2)\,, \qquad \tilde{g}_{AT} = \frac{4F_V d_3}{F_0}\,. \tag{5.40}$$

The value of d_3 may be determined by enforcing asymptotic conditions concerning the $\langle VVP\rangle$ correlator. Not all conditions can be satisfied at the same time if one uses a minimal resonance model [42, 43, 46]. If one chooses to enforce the overall scaling behaviour one obtains [41]

$$d_3^{\text{scaling}} = \frac{F_0^2}{8M_V^2} - \frac{N_c M_V^2}{64\pi^2 F_V^2}\,. \tag{5.41}$$

Numerically, this gives $\tilde{g}_{AT} = -0.58$ which is somewhat smaller in magnitude than $\tilde{g}_V$. Alternatively, one may choose instead to impose that the $\langle VVP\rangle$ correlator satisfies the VMD property, in which case one gets

$$d_3^{VMD} = -\frac{N_c M_V^2}{64\pi^2 F_V^2}\,. \tag{5.42}$$

If one uses this latter determination, one gets $\tilde{g}_{AT} \simeq -0.93$, i.e. the result from the antisymmetric tensor formalism becomes essentially identical to the one from the vector formalism.

6. Numerical results

6.1 Results for $K_1, \ldots, K_6$

In the numerical applications we will simply use $M_V = 0.77\,\text{GeV}$ and $F_V = \sqrt{2}F_\pi$ (KSRF relation [47]), which corresponds to $z = 2$ (the same values were used by BU). We will also

$10^3\,K_1^r$	$10^3\,K_2^r$	$10^3\,K_3^r$	$10^3\,K_4^r$	$10^3\,K_5^r$	$10^3\,K_6^r$	
−4.44	−1.73	4.44	−3.46	13.31	5.19	$\rho,\ a_1$
−6.8	−2.7	6.8	−5.5	20.3	8.2	$\rho,\ a_1$ (BU)
1.97	1.97	−1.97	3.93	−1.97	−1.97	$\omega\rho\pi$
−0.35	0.35	0.35	0.69	0.35	−0.35	scalars
0.4	−0.4	−0.4	−0.7	−0.4	0.4	scalars (BU)
0.11	0.11	−0.11	0.21	−0.11	−0.11	$b_1(1235)$
−2.71	0.69	2.71	1.38	11.59	2.77	Total

Table 1: Numerical results obtained for $K_i^r(\mu)$ with $\mu = 0.77\,\mathrm{GeV}$ from the present work. The contributions associated with the $b_1(1235)$ are discussed in appendix B. The results from ref. [29] are also shown for comparison.

use $F_0 = F_\pi = 92.4\,\mathrm{MeV}$. The results for $K_1^r(\mu), \ldots, K_6^r(\mu)$ with $\mu = M_V$ are collected in table 1. We show separately the contributions due to the scalars (terms proportional to c_d, with $c_d = 32\,\mathrm{MeV}$) and the contributions from the $\omega\rho\pi$ interactions (terms proportional to $\tilde{g}_{AT}$, with $|\tilde{g}_{AT}| = 0.93$). We find that this contribution can be significant for some of the couplings like K_2^r or K_4^r. The contributions related to the scalars are rather small. In that case, our results agree on the magnitude but have a different sign from those of BU because of the extra contribution needed to ensure consistency. The largest contributions are those originating from the $a_1\rho\pi$ sector. In that case our numerical results are in qualitative agreement with those of BU. We have also performed an estimate of the contributions associated with the b_1 meson (see appendix B) which turn out to be rather small.

6.2 An update on the corrections to Dashen's theorem

A reliable evaluation of the electromagnetic part of the $K^+ - K^0$ mass difference is important for the determination of $m_u - m_d$. At leading order in the chiral expansion it is given by Dashen's theorem [26]

$$(\Delta M_K^2)_{EM} = (\Delta M_\pi^2)_{EM} + O(e^2 m_q)\,. \tag{6.1}$$

It was pointed out that the corrections of order $e^2 m_s$ could be rather large [48] such that the right hand side could be modified by as much as a factor of two. Using Urech's formalism one can compute explicitly these corrections which have the following form [4]

$$(\Delta M_K^2)_{EM} - (\Delta M_\pi^2)_{EM} = e^2 M_K^2\,(A_1 + A_2 + A_3) + O(e^2 M_\pi^2) \tag{6.2}$$

with

$$\begin{aligned} A_1 &= -\frac{1}{16\pi^2}\left[(3+2Z)\log\frac{M_K^2}{\mu^2} - 4\right] - 16 Z L_5^r\,, \qquad Z = \frac{C}{F_0^4} \\ A_2 &= -\frac{4}{3}(K_5^r + K_6^r) \\ A_3 &= 8(K_{10}^r + K_{11}^r)\,. \end{aligned} \tag{6.3}$$

As discussed in (I) (and also in [30]) the Urech couplings K_{10}^r, K_{11}^r taken separately suffer from short distance ambiguities and depend on the gauge. The sum, however, is perfectly well defined and can be estimated from sum rules related to the correlators $\langle V^3V^3 - V^8V^8 - A^3A^3 + A^8A^8\rangle$ and $\langle VAP\rangle$, giving the following result [31]

$$
\begin{aligned}
K^r_{10} + K^r_{11} \;=\; \frac{3}{16}\frac{1}{16\pi^2}\Bigg\{ &\log\frac{M_V^2}{\mu^2} + Z_A - Z_V + \\
&+ 2\log z\left[\frac{(z+1)^2}{2(z-1)^3} + 1\right] + \frac{2z(z-3)}{(z-1)^2} + \frac{1}{6} + 4Z^0_\mu\Bigg\}
\end{aligned}
\tag{6.4}
$$

with

$$
Z_A - Z_V = \frac{4M_A^2(F_\pi - F_K)}{F_\pi(M_\pi^2 - M_K^2)} + \frac{2F_V^2 M_\rho(M_\rho - M_\phi)}{F_\pi^2(M_\pi^2 - M_K^2)}\log z - 2\log z - 2\,. \tag{6.5}
$$

The quantity $Z_A - Z_V$ is generated from flavour symmetry breaking at first order in the vector multiplet as well as the axial-vector multiplet and the computation takes into account Weinberg-type sum rule relations for the splittings of the masses as well as the coupling constants. The last entry in the expression (6.4), Z^0_μ, is a contribution which should restore the scale dependence proportional to C/F_0^4. Since this effect is subleading in N_c, this term will be dropped here for consistency with previous approximations. Making use of the result eq. (6.4) and of the numerical values for $K^r_5 + K^r_6$ shown in table 1 we find the following numerical values for the three corrective terms (which we give in units of the physical $\pi^+ - \pi^0$ mass difference, $\Delta M_\pi^2 = 1261.16\,\mathrm{MeV}^2$)

$$
(\Delta M_K^2)_{EM} - (\Delta M_\pi^2)_{EM} = \Delta M_\pi^2\,[0.59 - 0.34 + 1.25]\,. \tag{6.6}
$$

The new result with respect to (I) concerns $K_5 + K_6$ and we find that this contribution goes in the sense of reducing the size of the correction. This is in qualitative agreement with the result of BU. However, the effect of the extra terms from $\mathcal{L}_{a_1\rho\pi}$ included here is to reduce the magnitude of $K_5 + K_6$ by approximately a factor of two as compared to BU. There is a clear indication for a sizable correction to Dashen's theorem rather similar to that found in refs. [49, 30]. The origin of the different result obtained in ref. [29] can be traced, essentially, to the fact that their result for $K_{10} + K_{11}$ is ten times smaller than ours. This is due to two reasons 1) flavour symmetry breaking in the vector and the axial-vector multiplets was ignored in [29] such that K_{10} was just set to zero and 2) the effect of the couplings from $\mathcal{L}_{a_1\rho\pi}$ which they did not take into account is to strongly enhance the value of K_{11}.

6.3 Uncertainties

One should keep in mind that there are uncertainties which are difficult to estimate quantitatively. These have two main sources. The first one is that we have used, at several places, large N_c approximations a) in the treatment of the resonances, which are taken to be infinitely narrow and b) in the calculation of QCD n-point correlators in which only tree graphs were considered. The second source of uncertainty is that we have retained the contributions from the lightest resonances only. The usual experience with such approximations is that reasonable order of magnitudes should be obtained.

7. Conclusions

In this paper, we have considered a set of sum rules for the electromagnetic chiral parameters which involve QCD 4-point correlators in connection with the construction of a chiral lagrangian with resonances. For our purposes, the resonance lagrangian provides rational-type approximations for the QCD correlators obeying the chiral Ward identities.

We have shown that the resonance lagrangian of order p^4 was not sufficient to ensure all the convergence conditions required by the sum rules. We have then shown that introducing a set of terms of order p^6 involving $a_1\rho\pi$-type couplings renders it possible to satisfy these conditions in the sense of finiteness as a $d \to 4$ limit. The new coupling constants which appear were shown to obey non linear equations, we have argued that physical requirements select a unique solution.

Our investigation, however, was essentially limited to a set of terms with three building blocks. A more general investigation should be performed in the future. A more complete determination of the resonance lagrangian would lead to much better estimates of the chiral coupling constants at this order [50] than those available at present. This would improve significantly the effectiveness of the chiral calculations at order p^6. There is also a limitation in the set of resonances which were included. In this regard, the role of tensor mesons should perhaps be investigated [51].

We have considered an application to the problem of the chiral corrections to Dashen's low-energy theorem. We have found that the combination $K_5^r + K_6^r$ as determined here reduces the size of the correction which still remains rather large. One motivation for deriving estimates for each of the parameters K_i is to improve the accuracy of radiative correction evaluations. In this context, further work is still needed because another set of parameters, X_i, is involved in the interesting case of the semi-leptonic decays [5]. Finally, a formal similarity can be noted between the sum rules considered here and those shown recently to hold for chiral parameters in the weak non-leptonic sector [52].

Acknowledgments

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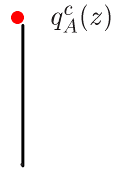
A. Vertices

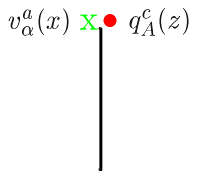
We list below the set of vertices which are needed in section2 and those needed section3 which are associated with vector and axial-vector resonances. The vertices represent derivatives of the action (times i) with respect to the various sources (with each source multiplied by i). It is not difficult to derive, in an analogous way, all the other relevant vertices.

A.1 Vertices with pions

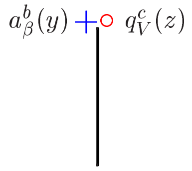
$$V = -F\partial_\lambda \pi^c(z) F^\lambda(z)$$



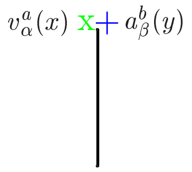
$$V = iFf^{cat}\pi^t(x)F_\alpha(x)\delta(x-z)$$



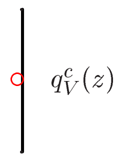
$$V = iFf^{bct}\pi^t(y)F_\beta(y)\delta(y-z)$$



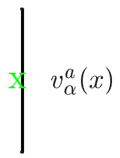
$$V = iFf^{bat}\pi^t(x)g^{\alpha\beta}\delta(x-y)$$



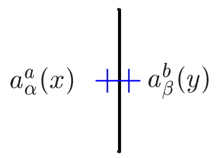
$$V = -f^{stc}\partial_\mu \pi^s(z)\pi^t(z)F_\mu(z)$$



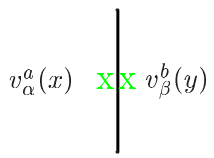
$$V = -f^{sta}\partial_\alpha \pi^s(x)\pi^t(x)$$



$$V = i\, g^{\alpha\beta} f^{ais} f^{bjs} \pi^i(x)\pi^j(x)\delta(x-y)$$



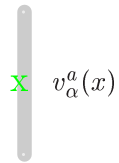
$$V = -i\, g^{\alpha\beta} f^{ais} f^{bjs} \pi^i(x)\pi^j(x)\delta(x-y)$$



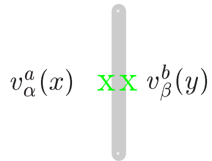
A.2 Resonance vertices with v_μ sources

These vertices are the same for vector or axial-vector mesons:

$$V = -f^{aij} R^i_{\alpha\mu}(x) \partial^\lambda R^{j\mu}_\lambda(x)$$

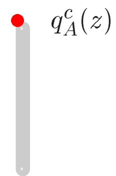


$$V = i f^{ail} f^{bjl} R^i_{\alpha\mu}(x) R^{j\mu}_\beta(x) \delta(x-y)$$

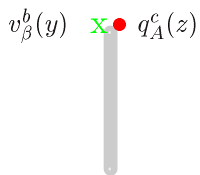


A.3 Vertices proportional to F_A

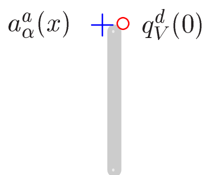
$$V = -F_A A^c_{\mu\nu}(z) \partial^\mu F^\nu(z)$$



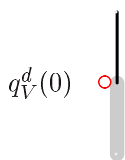
$$V = i F_A f^{ibc} A^i_{\beta\lambda}(y) F^\lambda(y) \delta(y-z)$$



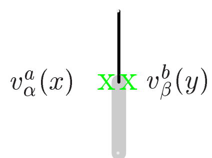
$$V = i F_A f^{iad} A^i_{\alpha\lambda}(x) F^\lambda(x) \delta(x)$$



$$V = \frac{-F_A}{F} f^{dij} A^i_{\mu\nu}(0) \partial^\mu F^\nu(0) \pi^j(0)$$



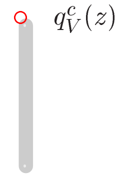
$$V = \frac{F_A}{F} f^{ijl} f^{abl} A^i_{\alpha\beta}(x) \pi^j(x) \delta(x-y)$$



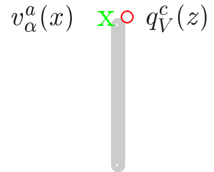
Note that this vertex is antisymmetric in the Lorentz indices $\alpha\beta$ and will therefore not contribute here.

A.4 Vertices proportional to F_V

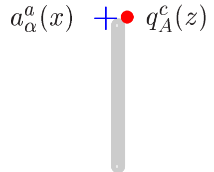
$$V = F_V V^c_{\mu\nu}(z)\partial^\mu F^\nu(z)$$



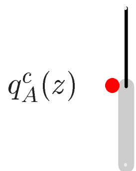
$$V = -iF_V f^{iac} V^i_{\alpha\lambda}(x) F^\lambda(x)\delta(x-z)$$



$$V = -iF_V f^{iac} V^i_{\alpha\lambda}(x) F^\lambda(x)\delta(x-z)$$

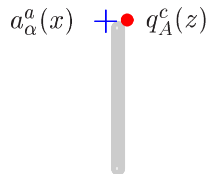


$$V = \frac{F_V}{F} f^{stc} V^s_{\mu\nu}\partial^\mu F^\nu \pi^t$$

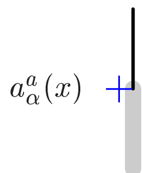


A.5 Vertices proportional to G_V

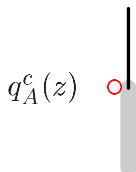
$$V = 2iG_V f^{act} V^t_{\alpha\nu}(x) F^\nu(x)\delta(x-z)$$



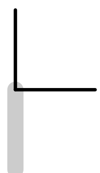
$$V = \frac{-2G_V}{F} f^{ast} V^s_{\alpha\mu}(x)\partial^\mu \pi^t(x)$$



$$V = \frac{-2G_V}{F} f^{cst} V^s_{\lambda\mu}(z)\partial^\mu \pi^t(z) F^\lambda(z)$$



$$V = \frac{-iG_V}{F_0^2} f^{stu} \int dv\, V^s_{\mu\nu}(v)\partial^\mu \pi^t(v)\partial^\nu \pi^u(v)$$



There are other potentially relevant vertices which are antisymmetric in $\alpha\beta$ and will thus not contribute for our purposes.

JHEP06(2004)047

B. $b_1(1235)$ contributions

We present here an estimate of the contributions associated with the $b_1(1235)$ resonance. The physics of the b_1 meson is not as constrained as that of the a_1 meson by chiral symmetry, so in this section we will occasionally make some educated guesses. The order of magnitude of the b_1 effect should nevertheless be properly accounted for. The contributions are to the same correlator components as those from the $\omega\rho\pi$ couplings. In principle, the convergence constraints should be applied to the sum. We will apply,however, these constraints separately to the b_1 for the purpose of obtaining an order of magnitude estimate.

We can write down a set of vertices in exact analogy with the $a_1\rho\pi$ vertices

$$\begin{aligned}\mathcal{L}_{b_1\omega\pi} =& (G_1^B g^{\mu_0\mu_1}g^{\mu_2\mu_3} + G_2^B g^{\mu_0\mu_2}g^{\mu_3\mu_1} + G_3^B g^{\mu_0\mu_3}g^{\mu_1\mu_2})\langle\, \{\nabla_{\mu_0}V_{\mu_1\lambda}, B_{\mu_2}{}^{\lambda}\}u_{\mu_3}\rangle + \\ &+(H_1^B g^{\mu_0\mu_1}g^{\mu_2\mu_3} + H_2^B g^{\mu_0\mu_2}g^{\mu_3\mu_1} + H_3^B g^{\mu_0\mu_3}g^{\mu_1\mu_2})\langle\, \{V_{\mu_1\lambda}, B_{\mu_2}{}^{\lambda}\}\nabla_{\mu_0}u_{\mu_3}\rangle\,. \end{aligned}\tag{B.1}$$

The only difference with the lagrangian of eq. (5.3) is that the commutator is replaced by an anticommutator on account of the fact that the b_1 meson is odd under charge conjugation. As a consequence, the contribution to the correlator $\langle AAQ_VQ_V\rangle_3$ can be deduced simply from the results of section 5.4

$$\begin{aligned}\langle AAQ_VQ_V\rangle_3^{b_1\omega\pi} =& \frac{-3F_V^2}{16\pi^2}\Bigg\{-(G_1^B+G_2^B+2G_3^B)^2\left[\log\frac{M_V^2}{\Lambda^2} + \frac{M_B^4\log\frac{M_B^2}{M_V^2}}{(M_B^2-M_V^2)^2} + \frac{7M_V^2-M_B^2}{6(M_V^2-M_B^2)}\right] + \\ &+4\frac{(G_1^B)^2M_V^2}{M_B^2}\left(\log\frac{M_V^2}{\Lambda^2}+\frac{5}{3}\right)\Bigg\}\,.\end{aligned}\tag{B.2}$$

Imposing a consistency condition on eq. (B.2) enables one to express $\langle AAQ_VQ_V\rangle_3^{b_1\omega\pi}$ in terms of the combination

$$\tilde{G}^B \equiv G_1^B + G_2^B + 2G_3^B \tag{B.3}$$

and one obtains

$$\langle AAQ_VQ_V\rangle_3^{b_1\omega\pi} = \frac{3F_V^2}{16\pi^2}\left(\tilde{G}^B\right)^2\left[\frac{M_B^4\log\frac{M_B^2}{M_V^2}}{(M_B^2-M_V^2)^2} + \frac{3M_B^2+M_V^2}{2(M_B^2-M_V^2)}\right]. \tag{B.4}$$

We must next try to determine the relevant combination of couplings $\tilde{G}^B$ from experiment. The widths of the decays $b_1^+ \to \gamma\pi^+$ and $b_1^+ \to \omega\pi^+$ are known

$$\Gamma_{b_1^+\to\gamma\pi^+} = 227 \pm 59\,\mathrm{KeV}\,, \qquad \Gamma_{b_1^+\to\omega\pi^+} = 142 \pm 9\,\mathrm{MeV} \tag{B.5}$$

and their expressions in terms of the resonance lagrangian coupling constants read

$$\begin{aligned}\Gamma_{b_1^+\to\gamma\pi^+} &= \frac{\alpha(M_B^2-M_\pi^2)}{24M_B}\left(\frac{F_VM_B^2}{3F_\pi M_V^2}\right)^2\left(\tilde{H}^B\right)^2 \\ \Gamma_{b_1^+\to\omega\pi^+} &= \frac{p_{cm}(M_B^2-M_\omega^2)^2}{48\pi M_B^4M_\omega^2F_\pi^2}\left\{2M_B^2M_V^2\left(\tilde{G}^B\right)^2 + \left[(M_B^2-M_V^2)\tilde{H}^B + 2M_V^2\tilde{G}^B\right]^2\right\}.\end{aligned}\tag{B.6}$$

Here, we have introduced the notation

$$\tilde{H}^B \equiv H_1^B + H_2^B + G_2^B + 2G_3^B \,. \tag{B.7}$$

Using these two pieces of information enables one to determine the combination of couplings that one needs, but there are several solutions due to the non linearity of the equations

$$\tilde{H}^B = \pm 0.68 \,, \qquad \tilde{G}^B \equiv G_1^B + G_2^B + 2G_3^B = \pm 0.24, \ \pm 0.67 \,. \tag{B.8}$$

We can eliminate some of the solutions by using one more piece of experimental information concerning the D/S ratio in the decay $b_1 \to \omega\pi$

$$\frac{f_D}{f_S} = 0.29 \pm 0.04 \tag{B.9}$$

The S and the D-wave components of the decay amplitude $b_1 \to \omega\pi$ (defined as in ref. [53]) are easily computed from our lagrangian

$$\begin{aligned} f_S &= -\frac{\sqrt{4\pi}}{3M_B M_\omega F_\pi}\left(M_B^2 - M_V^2\right)\left[M_V(M_B + 2M_V)\tilde{G}^B + (M_B^2 - M_V^2)\tilde{H}^B\right] \\ f_D &= -\frac{\sqrt{8\pi}}{6M_B M_\omega F_\pi}\left(M_B^2 - M_V^2\right)\left[2M_V(M_B - M_V)\tilde{G}^B + (M_B^2 - M_V^2)\tilde{H}^B\right] . \end{aligned} \tag{B.10}$$

Using this result and the experimental one on the D/S ratio reduces the set of acceptable solutions for the coupling constants to

$$(\tilde{G}^B, \tilde{H}^B) = \pm(0.24, 0.68) \tag{B.11}$$

which give $f_D/f_S = 0.28$.

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